Solutiont to the Welcome "Attendance Quiz" for Dr. Z.'s June 30, 2026 DIMACS REU Lecture

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1. Who was (or is) Paul Erdos?

The greatest mathematician of all time (even greater than Archimedes, Euler, and Gauss)

2. Define "Erdos Number".

The Erdos number of X is the length of the shortest path from X to Paul Erdos in the collaboration graph. E.G. $EN(Erdos) = 0$, $EN(NogaAlon) = 1$ (and at least 500 others). $EN(DrZ) = 2$ (and at least 5000 others), $EN(LazarosGallos) = 3$ (BTW $LazarosNumber(Dr.Z) = 4$)

3. Define the number two.

Peano: The successor of 1 .

von Neumann: $\{\emptyset, \{\emptyset\}\}$.

Frege: The equivalence class (under 'being in bijection') of **all** sets with two elements.

Dr. Z. : The equivalence class (under 'being in bijection') of **all** two-element subsets of the **finite** set of all positive integers.

4. Define the number e

This was a trick question! e is not a number! Not even $\sqrt{2}$ is a number. The only numbers that exist are quotients of integers, aka *rational numbers*) and again there is only a finite supply of them (but it is SO big that we will never have to worry about having an overflow in the 'big computer in the sky'). e (and π, and even $\sqrt{2}$) are **equivalence classes** of algorithms that output finite sequences of rational numbers whose differences go to 0 (you can phrase in fully constructive finitistic terms).

5. Define the number π

See above, two 'algorithms' (mentioned by the students) are $4(1 - 1/3 + 1/5 - 1/7 + \ldots)$ and $\sqrt{6(1/1^2 + 1/2^2 + 1/3^2 + \ldots)}$

The definition *ratio of the circumference of a circle to its diameter* is flawed, since it tacitly assume that real numbers exist and that the mathematical universe is continuous. You can have a version of it that is OK, but I am too lazy to state it.

6. Define the Catalan number $C(n)$. Name at least one combinatorial family that it counts.

$C(n) := (2n)!/(n!(n+1)!)$. They count at least 500 non-trivially equivalent combinatorial families, the most important ones are in Richard Stanley's book 'Catalan Numbers'. One of them is the number of Dyck words of semi-length n (lists whose members are in $\{1, -1\}$ that add up to 0 and whose partial sums are non-negative). Also the number of (complete) binary trees with $n+1$ leaves (and hence n internal vertices), and many many other ones.