NAME: (print!) ____

E-Mail address: ____________________

MATH 477 (3), Dr. Z. , Exam 2, Monday, Nov. 27, 2017, 8:40-10:00am, HLL 116

PUT The FINAL ANSWER TO EACH PROBLEM IN THE AVAILABLE BOX

Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

EXPLAIN EVERYTHING! Only simple calculators are allowed. You need the Z table.

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1. (10 pts.) Let X, Y, Z be three random variables for which

$$Var(X) = 1 \quad , \quad Var(Y) = 1 \quad , \quad Var(Z) = 1 \quad ,$$

$$Var(X + Y) = 4 \quad , \quad Var(X + Z) = 4 \quad , \quad Var(Y + Z) = 4 \quad .$$

Find $Var(X + Y + Z)$

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ans.

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2. (10 points altogether)

(i) (3 points) State the formula for the expected number of rounds it takes for a gambler who enters a fair casino with x dollars, until she is is either broke or has N dollars. At each round she wins a dollar with probability $\frac{1}{2}$ and loses a dollar with probability $\frac{1}{2}$.

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ans.

(ii) (7 points) Give a complete proof, with all the details, of that formula. (You may use , w/o proof, the fact that the system of equations has a unique solution)

3. (10 points altogether)

(i) (3 points) State the formula for the expected number of coupons that a coupon-collector must buy before he gets a full collection, and there are N coupons altogether. It is assumed that at each purchase each of these coupons is equally likely to show up (but of course, you only see what you got after you open the package).

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ans.

(ii) (7 points) Using the fact that the Expectation of a Geometric random variable with parameter p is $\frac{1}{p}$, and the Geometric random variable is the number of trials until the first success, when you independently try something whose probability is p, prove the above formula.

4. (10 points) If you enter a casino with 800 dollars, and wish to make 1000 dollars, and the probability, at each round, of winning a dollar is 0.49 and losing a dollar is 0.51, what is the probability of exiting a loser?

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ans.

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5. (10 points) The joint density function of X and Y is given by

$$f(x, y) = \begin{cases} \frac{x+y}{5} & , \quad if \quad 0 < x < 2\,, 1 < y < 2\,; \\ 0 & , \quad otherwise, \end{cases}$$

If you know that $Y = 1.25$ what is the probability that $1 \leq X \leq 2$.

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ans.

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6. (10 points) In a certain community of married couples, the maximal income of the wife is 100K and the maximal income of the husband is also 100K. Every husband makes **at most** what his wife makes. Let X denote the the wife's income and let Y denote the husband's income. Let X and Y have joint density function $f(x, y) = 3\,(x^2 + y^2)$ on the region where the density is positive. The unit of money is 100K.
If it is known that the wife makes 50K dollars, what is the probability that the husband makes less that 25K?

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ans.

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7. (10 points) Let X and Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} 30x \quad , \quad for \quad x^4 \le y \le x^3\,; \\ 0 \quad , \quad otherwise. \end{cases}$$

Let g be the marginal density function of X. Find $g(x)$.

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ans.

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8. (10 points) A piece of equipment is being insured against early failure. The time from the purchase until failure of the equipment is exponentially distributed with mean 5 years. The insurance will pay an amount of $2A$ if the equipment fails during the first year, and it pays A if failure occurs during the second, third, or fourth year. If failure occurs after the first four years, no payment will be made.
At what level must A be set if the expected payment made under this insurance is to be 2000?

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ans.

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9. (10 points) Approximate the probability that if you toss a loaded coin, with $Pr(Head) = 0.6$, one thousand times , the number of times it lends Heads is ≥ 560 and ≤ 620.

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ans.

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10. (10 points)
Out of a class of 80 students,
• 20 students play football and soccer
• 20 students play football and basketball
• 20 students play soccer and basketball
• 10 students play football soccer and basketball
• 10 students play none of these three sports
• The number of students who play each of the three sports is the same
How many students play soccer?

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ans.
