We can iterate the contiguous relation `C55`

$$
{}_2F_1\left[\begin{matrix}a,b\\c\end{matrix};z\right]=\frac{(1-a+b)\,(c-1)}{(a-1)\,(1+b-c)}\,{}_2F_1\left[\begin{matrix}a-1,b\\c-1\end{matrix};z\right]
+\frac{b\,(c-a)}{(a-1)\,(c-b-1)}\,{}_2F_1\left[\begin{matrix}a-1,b+1\\c\end{matrix};z\right]
$$

from *HYP* to obtain the contiguous relation

$$
{}_2F_1\left[\begin{matrix}a,b\\c\end{matrix};z\right]=\sum_{k=0}^{m}\binom{m}{k}\frac{(c-m+k)_{m-k}\,(b-a+k+1)_{m-k}\,(b)_k\,(a-c-k+1)_k}{(a-m)_m\,(b-c+1)_m}
\cdot{}_2F_1\left[\begin{matrix}a-m,b+k\\c-m+k\end{matrix};z\right].
$$

If we apply this (with $m=r$) to

$$
{}_2F_1\left[\begin{matrix}b,-2n\\2r+2-2n-b\end{matrix};-1\right],
$$

then we obtain

$$
\sum_{k=0}^{r}\binom{r}{k}\frac{(r+2-2n-b+k)_{r-k}\,(-2n-b+k+1)_{r-k}\,(-2n)_k\,(2b-2r+2n-k-1)_k}{(b-r)_r\,(-2r+b-1)_r}
\cdot{}_2F_1\left[\begin{matrix}b-r,-2n+k\\r+2-2n-b+k\end{matrix};-1\right].
$$

Now the ${}_2F_1$-series is just by 1 “off” from Kummer’s summation formula. To arrive there exactly, we apply `C34` from *HYP*:

$$
{}_2F_1\left[\begin{matrix}a,b\\c\end{matrix};z\right]=\frac{(c-1)}{c-1-b}\,{}_2F_1\left[\begin{matrix}a,b\\c-1\end{matrix};z\right]+\frac{b}{1+b-c}\,{}_2F_1\left[\begin{matrix}a,b+1\\c\end{matrix};z\right].
$$

For the “odd” case, it suffices to apply the iteration of `C55`.