Attendance Quiz (PennState)

Sushmanth Jacob Akkarapakam

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1 Introduction

Problem. Solve the set of equations

$$\begin{aligned} x_1 + x_2 &= 2, \\ x_1 y_1 + x_2 y_2 &= 0, \\ x_1 y_1^2 + x_2 y_2^2 &= 2, \\ x_1 y_1^3 + x_2 y_2^3 &= 0. \end{aligned}$$

Solution. Let's take

$$\phi(\theta) = \frac{x_1}{1 - \theta y_1} + \frac{x_2}{1 - \theta y_2}.$$

Then we may suppose

$$\phi(\theta) = \frac{A_1 + A_2\theta}{1 + B_1\theta + B_2\theta^2} = 2 + 2\theta^2 + \cdots.$$

Cross-multiplying and equating coefficients gives us

$$\begin{aligned} A_1 &= 2, \\ A_2 &= 2B_1, \\ 0 &= 2B_2 + 2, \\ 0 &= 2B_1. \end{aligned}$$

Hence, we see that

$$\phi(\theta) = \frac{2}{1 - \theta^2} = \frac{1}{1 - \theta} + \frac{1}{1 + \theta},$$

thus giving us the solution as $(x_1, x_2, y_1, y_2) = (1,\ 1,\ 1, -1)$.