Proof of Lemma

Suppose

$$\frac{n_1(n_1+1)}{2}+\frac{n_2(n_2+1)}{2}\equiv(l-6)(24)^{-1} \pmod{l}.$$

Then

$$(2n_1+1)^2+(2n_2+1)^2\equiv(l-6)3^{-1}+2 \pmod{l}.$$

If $l=12m+7$ then $3^{-1}\equiv -4m-2$ and

$$(l-6)3^{-1}+2\equiv(12m+1)(-4m-2)+2=-48m^2-28m=-4ml\equiv 0 \pmod{l}$$

while if $l=12m+11$ then $3^{-1}\equiv 4m+4$ and

$$(l-6)3^{-1}+2\equiv(12m+5)(4m+4)+2=48m^2+68m+22=(4m+2)l\equiv 0 \pmod{l}.$$

In either case,

$$(2n_1+1)^2+(2n_2+1)^2\equiv 0 \pmod{l}.$$

Since $l\equiv 3 \pmod 4$, we have either $2n_1+1\equiv 0 \pmod{l}$ or $2n_2+1\equiv 0 \pmod{l}$.

Since $0\le n_1,\ n_2\le l-1,\ 1\le 2n_1+1,\ 2n_2+1\le 2l-1$, so either $2n_1+1=l$ or $2n_2+1=l$.

So either $n_1=\dfrac{l-1}{2}$ or $n_2=\dfrac{l-1}{2}$.

QED.