

Report on “The number of inversions ...”, by Baxter and Zeilberger

H. Wilf

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The purpose of this paper is to show that the number of inversions and the major index of a permutation are asymptotically jointly normally distributed. The paper is very well written, in an engaging style that captures the reader’s interest and doesn’t let go. There are two nice concepts that are introduced here, which personally I had not seen before, though that might reflect solely on my own lack of culture.

The first idea is that when we’re trying to find a formula for a certain combinatorial statistic, e.g., for n -permutations, we might carry out the following program: (a) show that the desired statistic is a polynomial in n , and bound its degree. Then, (b) compute enough numerical values of the statistic to determine the unknown coefficients of the polynomial. This avoids playing around with multiple sums, and it is attributed to [GKP] sec. 8.2, though I could not find it there.

If in fact this is not fully described in some accessible reference, then the authors should invest another paragraph of their paper in saying precisely and carefully why the covariance of maj and inv is a polynomial of degree 4 or less.

The second idea is that when we have a recursively defined counting function and we want to find its asymptotic behavior, one way to proceed is to show that for each n this function must be of a certain form, and therefore since the leading term of the asymptotics must satisfy the same recurrence as the function, we can determine this leading term directly from the recurrence. This is sufficient to prove the authors main result.

The paper should be published.