**How to play poker if you hate to lose?**

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**Abstract**:

Welcome to the world of poker, where strategy and probability rule. Picture yourself at the poker table, every decision a crucial step toward victory or defeat. Poker is not just a game of luck; it is a battlefield where strategy and probability rule. It was analyzed by the Hungarian-American mathematician John von Neumann, who believed that real life mirrors poker, involving bluffing and strategic thinking. Together with Oskar Morgenstern, he analyzed poker, resulting in their 1944 book *Theory of Games and Economic Behavior* [vNM], which laid the foundation for groundbreaking mathematical theory of economic and social organization.

**Simplified Von Neumann Poker**

**Part I: What is you don't mind Losing? (sometimes)**

In this paper, we consider a simplified von Neumann poker game, the payoff of which is summarized .... This model is the simplest because raising does not play a role, and Player 2 does not play. We will analyze a simplified model inspired by von Neumann, exploring a two-player scenario where each player antes, receives a random card, and decides whether to fold or call based on their hand.

Player 1 antes \$1 into the pot. Player 2 antes \$$(B + 1)$ into the pot. Each player is then dealt a random card between 1 and $n$. After seeing their own card, Player 1 can choose to "fold," in which case Player 2 wins the pot. Player 1 can alternately "call," which means to match the $B+1$ dollars to the pot. Then the two cards are compared, and the player with the higher number wins.

**Objectives**

Our goal is to find Player 1's optimal strategy and conduct mathematical experiments through Maple to understand its implications on concepts such as moments, coefficient of variation, and probability generating functions. We conclude the paper with a discussion of strategies for risk-averse players who prefer lower payoff uncertainty over higher uncertainty.

Let's shuffle up and deal as we explore the mathematics of poker, pioneered by the founder of Game Theory.

**Step-by-step procedure**

**First step:** Assume that the given card to Player 1 is $x$. The initial question to ask is "When to call?"

Player 1 will call (rather than fold) if

Thus, the strategy for Player 1 is to fold if the given card

**Notation:** From here on, for simplicity, we assume $\frac{Bn+B+2}{2(B+1)}$ is an integer, and we denote this cutoff integer value as $C$.

**Step 2:** "What is the expected payoff?"

Let $P_x$ be the random variable representing payoff of Player 1 under this strategy. Then,

Moreover, we obtain the following formulas:

• Since $C$ is an integer,

• When $B = 2$

**Step 3:** "What is the uncertainty?"

The variance of this play, if Player 1 follows this strategy, would be:

**Step 4:** "Higher moments?"

We calculate the higher moments about the mean:

**Step 5:** "Scaled moments as $n \to \infty$?"

This distribution is right-skewed as the third scaled moment, skewness, is positive.

**Generating Functions** Next we use the probability generating function to double-check our results and see if there is any closed-form solution.

**Remark:** Unfortunately, we can't find a nice closed-form for $f(t)$.

**References**

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**May 31, 2024** .