Solution of 12022. (Proposed by Mircea Merca), by Shalosh B. Ekhad,
ShaloshBEkhad@gmail.com

Let $F(n,k)$ be the summand. We have to prove that $a(n) := \sum_{k=0}^{n-1} F(n,k)$ equals n. We cleverly come up with $G(n,k) \,=\, -(1-x^n)\cdots(1-x^{n-k+1})$, that satisfies
$F(n+1,k) - F(n,k) = G(n,k+1) - G(n,k)$ (check!) . Hence

$$a(n+1)-a(n) = F(n+1,n)+\sum_{k=0}^{n-1}(\,F(n+1,k)-F(n,k)\,) = F(n+1,n)+\sum_{k=0}^{n-1}(\,G(n,k+1)-G(n,k)\,)$$

$$= \, F(n+1,n) \,+\, G(n,n) - G(n,0) \,=\, (1-x^n)\cdots(1-x) + (-(1-x^n)\cdots(1-x)) - (-1) \,=\, 1 \quad .$$

Since $a(0)=0$, we have $a(n)=n$. The second identity is obtained from the first by replacing x by $1/x$ and simplifying.