MATH 151 (17-19), Dr. Z. , **Practice For Midterm II**, First Posted: Nov. 14, 2004.

This version: Nov. 20, 7:30pm. Some typos have been corrected. The posted answers refer to *this version*. If you have an earlier version, please download the latest version.

For each kind of question, there is a *warm-up version*, that you should do first. The warm-up is too easy to be a typical exam question. This is followed by *moderate versions* and *hard versions*, both of which (kinds!, not the very same question) are possible exam questions.

1. (warm-up) Find the second derivative of (a) $f(x) = x^4 + 3x^2 + 4$, (b) $f(x) = 1/x^4$, (c) $f(x) = 4 \ln x$.

1. (moderate) Find the second derivative of (a) $f(t) = t \cos t$, (b) $f(t) = e^t - t^2 \sin t$, (c) $f(s) = s^2 e^s$, (d) $(x+1)/(x+3) + 2e^x$.

1. (hard) Find the second derivative of (a) $f(x) = 2 \sin^{-1}(x^3) + x^3$; (b) $\ln(\tan^{-1} x)$.

2. (warm-up) Sketch the function

$$f(x) = x^2 - 4x + 1 \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, (if applicable), intervals of increase and decrease, and concave up/down.

2. (medium) Sketch the function

$$f(x) = 2x^3 - 9x^2 + 12x + 1 \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, intervals of increase and decrease, and concave up/down.

2. (hard) Sketch the function

$$f(x) = 3x^4 - 4x^3 + 5 \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, intervals of increase and decrease, and concave up/down.

3. (warm-up-i) Sketch the curve

$$y = \frac{1}{x-1} \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, intervals of increase and decrease, and concave up/down.

3. (warm-up-ii) Sketch the curve

$$y = \frac{1}{(x-2)^2} \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, intervals of increase and decrease, and concave up/down.

3. (warm-up-iii) Sketch the curve

$$y = \frac{-1}{x+1} \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, intervals of increase and decrease, and concave up/down.

3. (moderate-i) Sketch the curve

$$y = \frac{x}{x-1} \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, intervals of increase and decrease, and concave up/down.

3. (moderate-ii) Sketch the curve

$$y = \frac{2x}{x+1} \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, intervals of increase and decrease, and concave up/down.

3. (hard-i) Sketch the curve

$$y = \frac{x}{x^2-4} \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, intervals of increase and decrease, and concave up/down.

3. (hard-ii) Sketch the curve

$$y = xe^{-2x} \quad ,$$

indicating all asymptotes (if applicable), local extrema, inflection points, intervals of increase and decrease, and concave up/down.

4. (war-up) If two resistors with resistances R_1, R_2 are connected in series then the total resistance R, measured in ohms (Ω), is given by

$$R = R_1 + R_2$$

If R_1, R_2 are decreasing at rates of $2\ \Omega/s$, $3\ \Omega/s$, respectively, how fast is R changing when $R_1 = 2\ \Omega$ and $R_2 = 3\ \Omega$.

4. (moderate-i) If three resistors with resistances R_1, R_2 R_3 are connected in parallel then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If R_1, R_2, R_3 are decreasing at rates of $2\ \Omega/s$, $3\ \Omega/s$, $5\ \Omega/s$ respectively, how fast is R changing when $R_1 = 1\ \Omega$ and $R_2 = 1/2\ \Omega$. $R_3 = 1/3\ \Omega$.

4. (moderate-ii) The volume of a spherical ball is expanding at a rate of $400\pi\, cm^2/s$. How fast is the radius changing when the radius is $10\, cm$?

4. (moderate-iii) The top of 15-ft ladder is sliding down a vertical wall at a rate of $1\, ft/sec$. How fast is the bottom of the ladder moving away from the wall when the top is 12 ft above the floor.

4. (hard) A water trough is of length 30 meters and a cross-section has the shape of an isosceles trapezoid that has width 10 meters at the bottom, 20 meters at the top, and height 5 meters. If the trough is being filled with water at a rate of $10\, m^3/min.$, how fast is the water level rising when the water had depth 3 ?

5. (warm-up) Use differentials to estimate 1.01^2.

5. (moderate-i) Use differentials to estimate $\sin(\frac{\pi}{4} + .01)$.

5. (moderate-ii) Use differentials to estimate $82^{1/4}$.

5. (moderate-iii) Use differentials to estimate $\ln(e^2 + .03)$.

6. (warm-up) Use one step of Newton's method to find an approximation for the root of the equation $x^2 - 3 = 0$, taking $x_1 = 1$.

6. (moderate-i) Use two steps of Newton's method to find an approximation for the root of the equation $x^3 = 3x - 4$, taking $x_1 = 0$.

6. (moderate-ii) Use two steps of Newton's method to find an approximation for the root of the equation $x^5 = x^4 + 15$, taking $x_1 = 2$.

7. (warm-up) Verify that the function

$$f(x) = x^2$$

satisfies the hypothesis of the Mean Value Theorem on the closed interval $[0, 1]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

7. (moderate) Verify that the function

$$f(x) = x^5 + 1$$

satisfies the hypothesis of the Mean Value Theorem on the closed interval $[0, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

7. (hard) Verify that the function

$$f(x) = \ln x$$

satisfies the hypothesis of the Mean Value Theorem on the closed interval $[e^2, e^3]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

8. (warm-up) Find the absolute maximum and minimum values of $f(x) = x^2 + 1$ on the interval $[-1, 1]$.

8. (moderate) Find the absolute maximum and minimum values of $f(x) = 4x^5 - 5x^4 + 1$ on the interval $[0, 2]$.

9. (warm-up-i) Find two numbers whose sum is 20 and whose product is as big as possible.

9. (warm-up-ii) You have 100 feet of fence and you have to enclose a rectangular plot bordering a straight river (no fence along the river). What is the largest possible area that you can achieve, and what are the dimensions?

9. (medium-i) A closed box, with square base, is made to have a volume of $1000\,cm^3$. The material for the bottom and top is twice as expensive as the material for the sides. What are the dimensions that will minimize the total cost of the material?

9. (medium-ii) Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(\frac{3}{2}, 0)$.

9. (hard) What are the radius and height of a cylinder whose volume is 300π cubic inches, that minimizes the surface area.

9. (hard) An open box, with square base, is made to have a volume of $1000\,cm^3$. The material for the bottom costs \$1 per square meter, the material for the front and back costs \$2 per square meter the material for the left side costs \$3 per square meter the material for the right side costs \$4 per square meter What are the dimensions that will minimize the total cost of the material?

10. (warm-up) Find the most general antiderivative

$$f(x) = 2x + 3$$

10. (moderate-i) Find the most general antiderivative

$$f(x) = \frac{x^3 + 1}{x^2}$$

10. (moderate-ii) Find the most general antiderivative

$$f(x) = 3\sin x + 4\cos x + e^x$$

10. (hard-i) Find the most general antiderivative

$$f(x) = 3x + 5(1 - x^2)^{-1/2}$$

10. (hard-ii) Find the most general antiderivative

$$f(x) = 3x + \frac{5}{1 + x^2}$$

11. (warm-up) Find f if $f'(x) = 2x$, $f(1) = 1$.

11. (moderate-i) Find f if $f'(x) = 4x - 6/x^4, x > 0, f(1) = 6$.

11. (moderate-ii) Find f if $f'(x) = \sin x, x > 0, f(\pi/2) = 3$.

11. (hard-i) Find f if $f''(x) = x^2 + 1$, $f(1) = 1, f'(1) = 2$.

11. (hard-ii) Find f if $f'''(x) = x - 1$, $f(1) = 1, f'(1) = 2, f''(1) = 2$.