The Sum of Two and Seven and The Product of Three by Three

Semaj Srelles

(To appear in Journal of Integers (JI) [accepted by J. Rodan, editor-in-chief])

Abstract

In this brief note, we prove a result which was "accidently" found thanks to Lien Enaols's Online Encyclopedia of Integers. Namely, we prove by elementary techniques that the number that you get when you add the integers two and seven equals the number that you get when you multiply the integer three by itself.

1 Introduction

Recently I received an electronic mail message (from Knarf Namrelle) in which I was notified that a pair of duplicate entries existed in Lien Enaols's On-Line Encyclopedia of Integers. This involved entry 285100A, and the former entry 367300A. One of these entries was described as the outcome of *adding* the integers two and seven (i.e. $2+7$, in technical jargon), while the other was the *product* of the integer three by itself (in other words 3×3).

I soon notified Lien Enaols of this fact and he promptly combined the two entries into one entry, 285100A . Upon combining these two entries, he also noted that it was not officially a theorem (that adding two and seven *always* equals the product of three by itself), but that it was "certainly true".

The goal of this short note is to prove this result so that its status is indeed a theorem. We close the paper by noting two other results relating addition and multiplication.

2 The Results

Theorem 2.1 The outcome of adding the integer two and the integer seven equals the outcome of multiplying the integer three by itself.

Proof. In *base-one* (unary)

$$2 = 11 \quad ,$$

$$7 = 1111111 \quad .$$

Hence

$$2 + 7 = 11 + 1111111 = 111111111 \quad . \tag{1}$$

Also

$$3 = 111 \quad ,$$

Hence (recall that 3a=a+a+a)

$$3 \times 3 = 3 + 3 + 3 = 111 + 111 + 111 = 111111111 \quad . \tag{2}$$

Since the right sides of Eqs. (1) and (2) are the same, so are their left-sides (by the well-known *transitivity* of the $=$ relation). $\square$

We note that at least two other entries, in OEI, that came from addition, have elegant expressions as a product. We first consider the entry 577300A.

Theorem 2.2 The outcome of adding the integer one and the integer three equals the outcome of multiplying the integer two by itself.

Proof. Tautologically we have

$$1 = 1 \quad ,$$

and by definition (again, in *unary*)

$$3 = 111 \quad .$$

Hence

$$1 + 3 = 1 + 111 = 1111 \quad . \tag{3}$$

Also

$$2 = 11 \quad ,$$

Hence (recall that 2a=a+a)

$$2 \times 2 = 2 + 2 = 11 + 11 = 1111 \quad . \tag{4}$$

Since the right sides of Eqs. (3) and (4) are the same, so are their left-sides (by the well-known *transitivity* of the $=$ relation). $\square$

Finally we consider the entry 074820A. This leads to the conjencture

Conjecture: The outcome of adding the integer seven and the integer nine equals the outcome of multiplying the integer four by itself.

We leave the proof of this (assuming it is true) to the reader, as the calculations involved herein would be straightforward but tedious.

3 Acknowledgements

The author gratefully acknowledges Knarf Namrelle for bring this problem to his attention and Rep Nakah Wodnul for valuable e-conversations. The author also thanks the referees for their helpful insights.