The COMBINATORIAL ASTROLOGY of Rabbi ABRAHAM IBN EZRA

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Rabbi Abraham Ben Meir Ibn Ezra (1089-1164), from Tudela, Spain, was one of the all-time greatest in the following categories of intellectual endeavor.

1) **Poet**: e.g.: *aha yarad/al sfrad/ra min hashamaim/ eini eini yorda mayim* [Aha befell, on Spain, evil from the heavens, my eye, my eye, water falling]).

2) **Philosopher**: he was an extreme pantheist and neo-Platonist, who influenced Spinoza in his abstract conception of God.

3) **Biblical exegist**: he introduced a critical and grammatical approach to biblical commentary, and was much more 'modern' than apologists like Rashi. I particularly like his commentary on Leviticus 18:20, where he said that the sexual act can be divided into three kinds: the good kind, for procreation, the bad one, to satisfy beastly lust, and a neutral one, 'to alleviate the weariness of the body'.

4) **Grammarian**: by writing in Hebrew, he revealed to European scholars, who could not read the Arabic literature, the ideas of his predecessors, including the concept of the three-letter *shoresh* (root), and the method of *pealim* (verbs, actions).

5) **Mathematician**: e.g. he popularized Zero (that he called *galgal*(wheel)), and wrote a very influential textbook, 'The Book of Number', where among many other things, he showed how to reduce multiplication to taking squares by using $(a+b)(a-b) = a^2 - b^2$, and how to take squares by using the recursions: $(3a)^2 = 10a^2 - a^2, (n \pm 1)^2 = (n)^2 \pm 2n + 1$,

6) **Puzzler**: he (allegedly) saved his life and the life of his disciples, by solving the generalized Josephus problem (there were 15 good guys and 15 bad guys on a boat, when a storm started to rage, 15 passengers had to be thrown to the sea; how to arrange the 30 people in a circle, by drowning every ninth man, in such a way that the scoundrels all drown?).

But what he was probably most proud of was his **ASTROLOGY**. And indeed, he was the greatest astrologer of his time, and probably in the 'top three' of all times (the two others being Ptolemy and Kepler). In fact, a large part of his mathematics was inspired by applications to astrology.

One of his astrological treatises is called *Sefer HaOlam* (The Book of the World). Written in a polemical style, its main message is to warn users against 'wrong' applications of astrology. Of

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course, like most scholars until 'modern' times, he was an ardent believer in astrology, but only when it is practiced correctly.

In particular he warned that all the astronomical tables predicting the times of planetary conjunctions are erroneous, because they assume uniform motion of the planets. He also made a very good point on the accumulation of errors, and for the need to account for experimental errors, and how unreasonable it is to extrapolate from ancient data. Hence, he only relied on astronomical observations made by contemporary 'sages of experiments'.

Ibn Ezra also knew how to compute the seventh row of what later became to be called Pascal's triangle. Except for the trivial $k = 0, 1$, he showed how to compute $\binom{7}{k}$ for $k \leq 7$.

The practical problem that inspired these calculations was to find the number of possible planetary conjunctions. As every educated person knows, there are exactly seven planets: Sun, Moon, Mercury, Venus, Mars, Jupiter, and Saturn. Whenever a subset of cardinality larger than one shows up at the same sign, this has great astrological significance. How many such events are possible?

Here is a translation, from Hebrew, of the first half of the relevant passage in Ibn Ezra's Sefer HaOlam. He called planets 'servants' (*meshartim*), which perhaps meant servants of God.

And the combinations are hundred and twenty. And thus you can know their number. [It is] Known that every calculation that adds from one to any number that one wills, you can obtain by its value [multiplied] by its half together with half of one, and here is an example, we wanted to know what is the sum of the numbers from one to twenty. We will multiply twenty by its half and half of one, and [we get] two hundreds and ten. And now we can start to know how many combinations [involving] two servants. And it is known that the number of servants is seven. And Saturn can combine with six other servants. And six by its half and the half of one is one and twenty. And thus is the number of combinations of twos. [Now] we wanted to know the number of combinations of threes. Here we put Saturn and Jupiter and one of the others, their number is five. We multiply five by two and a half and a half, and get fifteen ....

Ibn Ezra, then computes the number of conjunctions of three planets without Saturn, and repeats the same argument to get $\binom{5}{2}$, then, in turn, $\binom{4}{2}$, $\binom{3}{2}$, and $\binom{2}{2}$, totaling 35.

What Ibn Ezra is doing here is using the formulas (in our notation)

$$\binom{n}{2} = \sum_{i=1}^{n-1} i = (n-1)\left(\frac{n-1}{2} + \frac{1}{2}\right) \quad , \tag{1}$$

and

$$\binom{n}{k} = \sum_{m=k-1}^{n-1} \binom{m}{k-1} \tag{2}$$

.

For $k = 3, 4$, he uses (2) repeatedly until he can use (1). Translating his reasoning for the calculation of $\binom{7}{4}$ to our notation reads as follows.

$$\binom{7}{4} = \binom{6}{3} + \binom{5}{3} + \binom{4}{3} + \binom{3}{3} =$$

$$[\binom{5}{2} + \binom{4}{2} + \binom{3}{2} + \binom{2}{2}] + [\binom{4}{2} + \binom{3}{2} + \binom{2}{2}] + [\binom{3}{2} + \binom{2}{2}] + \binom{2}{2}] =$$

$$[4(4/2+1/2)+3(3/2+1/2)+2(2/2+1/2)+1(1/2+1/2)]+[3(3/2+1/2)+2(2/2+1/2)+1(1/2+1/2)]+$$

$$[2(2/2 + 1/2) + 1(1/2 + 1/2)] + 1(1/2 + 1/2) = 35 \quad .$$

For $k = 5, 6, 7$, Ibn Ezra only uses (2), without (1), invoking direct enumeration.

Morals

1) We are lucky to have modern notation. Ibn Ezra was obviously far smarter than any of us, yet he had to struggle so much because he lacked the right notation and the precise idea of induction. It was Rabbi Levi Ben Gerson, in 1321, who *rigorously proved* the explicit expressions for the binomial coefficients, and even he, almost two hundred years later, had to go through a verbal nightmare because he did not quite have modern algebraic notation. I believe that we are about to witness another notational revolution, inspired by programming languages, in which future proofs would be phrased. I am sure that our grandchildren will view our current style of writing mathematics and proofs using English or Spanish with the same bewilderment and mild amusement that we view Ibn Ezra and Levi Ben Gerson's mouthfuls.

2) Do not be superstitious about so-called superstitions. Not only Abraham Ibn Ezra and Levi Ben Gerson, but also Kepler and Newton, considered Astrology as a real science, not a pseudo-science. Who knows which of our current 'scientific' views will be considered superstition and hogwash by future generations? I have two candidates: the actual infinity, and the insistence on 'rigorous proof'.