A Proof of Morley's Theorem (From The (Devil's) Book)

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According to Erdös Pál, God(the SF) has a notebook that contains short and insightful proofs of all theorems. Don Newman([N]) has just found such a proof of Morley's([M]) theorem:

Theorem ([M]): If you trisect[2] each angle of an arbitrary triangle, and continue the trisection rays indefinitely, then the triangle formed from the intersections of these rays is equilateral.

According to Doron Zeilberger, the devil(SC(?)), also has a notebook, that contains purely routine, albeit often long, proofs of all theorems. In this note we give such a proof.

Proof: `maple <CR> read mephisto: morley(ta,tb);<CR> quit` □

For the sake of completeness here is file `mephisto`.

```
f:=proc(a,b):(a+b)/(1-a*b):end:      f2:=proc(a);normal(f(a,a)):end:      f3:=proc(a);normal(f(a,f2(a))):end:

morley:=proc(ta,tb): eq1:=y=x*ta: eq2:=y=(x-1)*(-tb): Eq1:=y=x*f3(ta): Eq2:=y=(x-1)*(-f3(tb)): sol:=solve(Eq1,Eq2,x,y):
Dx:=subs(sol,x):Dy:=subs(sol,y): sol:=solve(eq1,eq2,x,y): Ax:=subs(sol,x):Ay:=subs(sol,y): eq3:=y=x*f2(ta): eq4:=(y-
Dy)=(x-Dx)*f(f(f2(ta),-tb),sqrt(3)): sol:=solve(eq3,eq4,x,y): Bx:=subs(sol,x):By:=subs(sol,y): eq6:=y=-(x-1)*f2(tb):
eq5:=(y-Dy)=(x-Dx)*(-1)*f(f(f2(tb),-ta),sqrt(3)): sol:=solve(eq5,eq6,x,y): Cx:=subs(sol,x):Cy:=subs(sol,y): ABsq:=(Ax-
Bx)**2+(Ay-By)**2: ACsq:=(Ax-Cx)**2+(Ay-Cy)**2: BCsq:=(Bx-Cx)**2+ (By-Cy)**2: normal(ABsq-BCsq),normal(ACsq-
BCsq); end:
```

References

[M] F. Morley, *"Geometry"*, John Hopkins Univ. Press, 1904.

[N] Donald J. Newman, *A new proof of Morley's theorem*, Math. Intell., to appear.

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[2] In a lovely talk, delivered by Newman on 3/29/95, in the Temple number theory seminar, he said that he was attracted to this statement (for the last 50 years) because of the kick that he derived from the apparent(?) illicitude of the act of trisection.