The Number of Monomer-Dimer Tilings of an m by n Rectangle with a Fixed Proportion of Monomers

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Abstract: We use symbolic and numeric computations to study the asymptotic behavior of the number of monomer-dimer tilings of a strip of fixed width with a fixed proportion of monomers, and study the "entropy" as a function of the propotion, and try to extrapolate the date to the whole plane (for a strip of width infinity).

Introduction

The problem of enumerating dimer tilings is one of the most celebrated problems in both combinatorics and statistical physics, that was solved completely by Kasteleyan and Temperly & Fisher ([K][FT]). The corresponding problem for the *monomer-dimer* problem is wide open, and is believed to be **intractable**.

Nevetheless, it is possible to say *something* about it.

In this article we will study the following problem.

Fix a rational number $k = a/b$, between 0 and 1, and a positive integer m, how many ways can you tile an m by $2nb$ using dimers (dominoes, i.e. 2×1 and 1×2 tiles) and monomers (1×1 tiles), with *exactly* $2nbm \cdot k = nma$ dimers? Let's call that number $T_{k,m}(2nb)$

We are mainly interested in the *entropy*

$$\lambda_{2,m}(k) := \lim_{n \to \infty} \frac{\log T_{k,m}(2nb)}{2mnb} \quad .$$

We will compute if for many rational numbers k, and for $m \leq 8$, and look at the trend, and try to estimate the limiting function $\lambda_{2,\infty}(k) = \lambda_2(k)$.

Supporting Maple Package and Output

All the results in this article were obtained by the use of the Maple packages

- `http://www.math.rutgers.edu/~zeilberg/tokhniot/BDD.txt` ,

whose output files, along with links to diagrams, are available from the *front* of this article

`http://www.math.rutgers.edu/~zeilberg/mamarim/mamarimhtml/bdd.html` .

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References

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