Book Review of "Mathematics by Experiments" by J. Borwein and D. Bailey and "Experimentation in Mathematics" by J. Borwein, D. Bailey and R. Girgensohn

Reviewed by Doron ZEILBERGER [1]

Once upon a time, in Ancient Greece, science was platonic and *a priori*. The Sun revolved around the Earth in a perfect circle, because the circle is such a perfect figure, there were four elements, because four is such a nice number etc. Then came Bacon, Boyle, Galileo, Kepler, Lavoisier, Newton, and their buddies, and revolutionized science and made it *experimental* and *empirical*.

But math remained a priori and platonic to this day. Kant even went to excruciating lengths to 'show' that geometry, while *synthetic*, is nevertheless *a priori*. Sure, all mathematicians, great and small, conducted experiments (until recently, using paper-and-pencil), but kept their diaries and notebooks well-hidden in the closet.

But stand-by for a **paradigm-shift**. Thanks to Its Omnipotence, The Computer, math, that last stronghold of dear Plato, is becoming (overtly!) *experimental*, *a posteriori* and even *contingent*.

But what are poor pure mathematicians to do? Their professional *weltanschauung*, in other words, **philosophy**, and more importantly, *working habits*, in other words, **methodology**, never prepared them for serving this new silicon master. Some of them, like the conceptual genius Alexander Grothendieck even consider it (seriously!) the devil. But, while Grothendieck, and many other pure mathematicians, strongly dislike and mistrust the computer, there are already some that start to see the light. For example, the great non-commutative geometer, Alain Connes, in a recent talk, stated that his computer confirmed a certain conjecture of his for thirty special cases, and consequently he is absolutely certain that the general conjecture is correct.

Mathematicians who want to jump on this band-wagon (but unlike most band-wagons this one is *here to stay*), better read these two books! Traditionalists may get annoyed, since the authors don't make any bones about 'math by experiment' being truly a paradigm shift, and even dedicate a whole section to the Kuhnian notion of paradigm-shift, quoting Planck that the only way for it to be accepted is to be patient and wait for the old guard to die.

These are such fun books to read! Actually, calling them *books* does not do them justice. They have the liveliness and *feel* of great websites, with their bite-size fascinating factoids, and many human- and math- interest stories and gems. But do not get fooled by the light-hearted, immensely entertaining, style. You are going to learn more math (experimental or otherwise) than you ever did from any two single volumes. Not only would you know much more math, but you would *learn by osmosis*, how to become an experimental mathematician.

One of the many highlights is a detailed, behind-the-scenes, account of how the amazing Borwein-

[1] Department of Mathematics, Rutgers University (New Brunswick), Hill Center-Busch Campus, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019, USA. `zeilberg@math.rutgers.edu`
`http://www.math.rutgers.edu/~zeilberg/` . First version: Nov. 12, 2004. This version: Nov. 12, 2004.

Bailey-Plouffe formula for π

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i} \left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6} \right) \quad ,$$

was discovered. (By the way, the 'Bailey' is the same Bailey but the Borwein is Jonathan's brother Peter. Simon Plouffe, a latter-day Ramanujan, is the webmaster of the celebrated Inverse Symbolic Calculator site.)

The BBP formula allows one to compute the billion-and-first digit of π without computing the first billion digits. It was discovered by the aid of the so-called PSLQ algorithm of Helaman Ferguson, (who is also an 'experimental mathematician' in another sense, being a noted mathematical sculptor). Once discovered, the *proof* is an elementary calculus exercise, but the haystack of such formulas is infinitely large, and to find the 'just right' formula requires ingenious experimental mathematics, that the authors generously share with the readers.

There is also has a very interesting chapter on *normality*, that attempts to tackle the famous, notoriously difficult, problem of proving that the decimal (or any base) expansion of famous constants like e and π behave 'randomly'. Aside from some constructive-but-contrived numbers (the *Champernowne constant* $0.12\ldots891011\ldots9899100101102\ldots$ and natural-but-non-constructive numbers (like Chaitin's Ω), there are no known examples. Who knows? Perhaps the experimental approach outlined here will lead to the ultimate solution.

When you do experiments, serendipitous mistakes may lead to breakthroughs. In the second book, the authors described an 'electronic Petri dish', that was obtained by typing *infty* (the TeX symbol for infinity) rather than the correct *infinity*, in a Maple session, which Maple interpreted as a mere *symbol*, and gave an actual (unexpectedly symbolic!) answer. This lead to a beautiful conjecture that was latter proved by Gert Almkvist and Andrew Granville. While this particular discovery is not quite penicillin, we should expect, in the future, many more such serendipitous discoveries generated by 'errors'.

Like all successful accounts of new, rapidly-growing, areas, these books are going to be victims of their own success, since the further development that they are going to inspire, will render them obsolete very fast. I am sure that in five years these two lovely books will seem very naive, and all their 'controversial' pronouncements are destined to become true-but-trite. But then again, they have the making of classics, and they would always be fun to read, even if they would seem *quaint* rather than *avant-garde*.

But I said enough! Buy these books, and read them, if possible, from beginning to end. But even if you open any of them at any random page, and read just that page, you will most likely learn something new and fascinating, and potentially useful. If you get hooked enough to read them cover-to-cover, then you are ready to become a full-fledged experimental mathematician.