The Number of Ways of Walking in $x_1 \geq \ldots \geq x_k \geq 0$ for n Days, Starting and Ending at the Origin, Where at each Day you may either Stay in Place or Move One Unit in any Direction, Equals the Number of n-Cell Standard Young Tableaux with $\leq 2k+1$ Rows.

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Theorem: See title .

Proof: Consider the number of ways of walking in $x_1 \geq \ldots \geq x_k \geq 0$, from the origin back to the origin, in n days, without the lazy option, i.e. where at each day one is obliged to move one unit in any (legal) direction. The exponential generating function for this sequence is given by a certain determinant due, in more general form, to D. Grabiner and P. Magyar, and spelled-out, for our case, in Eq. (9) of `http://arxiv.org/pdf/math/05012230` (by Chen et. al). Obviously, the exponential generating function for the same quantity for lazy walkers, who sometimes rest, is e^t times that, since the latter is the binomial transform of the former.

Surprise! This is exactly Gessel's formula for the exponential generating function for the number of Standard Young Tableaux with n cells and $\leq 2k+1$ rows, spelled out in Eq. (24) of R. Stanley's paper `http://arxiv.org/pdf/math/050125035` . $\square$

It would be interesting to find a *nice* bijective proof.

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