Theorem 1: For all $n > 0$

$$\sum_{i=1}^{n} i \;=\; \frac{n(n+1)}{2} + \prod_{i=1}^{10^{1000000000000000000}} (n-i) \quad .$$

Empirical Proof: True for $0 < n \leq 10^{1000000000000000000}$ (check!), hence true for all n.

Theorem 2: For all $n > 0$

$$\frac{n}{10^{1000000000000000000}} < 1 \quad .$$

Empirical Proof: True for $0 < n < 10^{1000000000000000000}$ (check!), hence true for all n.

Definition: The Pisot sequence , $E(x, y)$ where $0 < x < y$ are integers, is defined by the following nonlinear recurrence:

$$a_0 = x \quad , \quad a_1 = y \quad ,$$

and, for $n > 1$,

$$a_n := \left\lfloor \frac{a_{n-1}^2}{a_{n-2}} + \frac{1}{2} \right\rfloor \quad ,$$

where, as usual, $\lfloor x \rfloor$ denotes the largest integer that is $\leq x$.