I Am Sorry, Richard Ehrenborg and Margie Readdy, About Your Two Conjectures, But One is FAMOUS, While The Other Is FALSE

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R. Ehrenborg and M. Readdy[ER] raised two conjecture. The first one was that a certain triple-sum indexed by permutations of length n equals $1!3!5!\cdots(2n-1)!$. I don't remember the details right now, but I immediately recognized it as equivalent to a special case of the celebrated Mehta Integral Identity (that was "open" between 1965 and 1980, until one time Freeman Dyson mentioned it to Atle Selberg, and the latter immediately deduced it from his (then obscure, but now famous) 1944 integral).

The second conjecture was more interesting. It claimed that there are exactly $k!$ Down-Up *involutions* of length $2k$. Richard checked it for $k = 1, \ldots, 5$. Alas, using my Maple package `EHRENBORG`, it turns out (type `nDUI(12);`), that there are only 716 Down-Up involutions of length 12, which is 4 short of 6!.

For the sake of completeness, here are the first 24 terms of the enumerating sequence for Down-Up involutions:

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[1, 1, 1, 2, 3, 6, 11, 24, 51, 120, 283, 716, 1833, 4948, 13561, 38788, 112745, 339676,
1039929, 3283876, 10532747, 34717276, 116158851, 398257012] .
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REFERENCE

[ER] Richard Ehrenborg and Margie Readdy, *private communication*, during lunch with Theodore Ehrenborg, at the Indian restaurant **Masala Grill**, Princeton, NJ, June 11, 2002.

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`http://www.math.rutgers.edu/~zeilberg/` . First version: July 2, 2002. Accompanied by the Maple package `EHRENBORG` available from `http://www.math.rutgers.edu/~zeilberg/programs.html` . Supported in part by the NSF. Exclusively published in the 'Personal Journal of Ekhad and Zeilberger'
(url: `http://www.math.rutgers.edu/~zeilberg/pj.html`.)