A Computer Algebra Approach to the Discrete Dirichlet Problem]A Computer Algebra Approach to the Discrete Diric

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The Gröbner Basis Algorithm is used to find closed form expressions for the generating functions of finite difference eq

*Introduction Solutions to the Dirichlet problem for (elliptic) partial difference equations are needed both in the solutio

In order to approximate solutions of boundary value problems in partial differential equations, one sets up a so-called f

In random walk problems, the value of interest is either the probability of getting from a fixed point to another point u

As far as we know, the only method that is used for obtaining exact solutions for such discrete Dirichlet problems is by

Here we propose a new method for solving such problems, that reduces the problem to a much smaller system of linear

Our method uses a very small part of Buchberger's celebrated method of Gröbner basis, namely taking the "normal for

We will illustrate our method by a detailed example: The two-dimensional gambler's ruin problem, for which no closed

*Gambler's Ruin Examples
with Generating Functions Consider a gambler who wins a dollar or loses a dollar with probability $\frac{1}{2}$. The gambler starts ou

Now consider a two-dimensional version of this game: the gambler has two piles of money, say i dollars and j shekels. T

Consider first the one-dimensional(pile) game. Let $f(i) = f_n(i)$ be the expected duration of the game. Then

Let

be the generating function for the problem. Then,

Now let $f(n-1) = f(1) = c$. Then

So,

Now F, we know, is a polynomial by definition. So it can be determined that $c = n-1$ by long division and setting th

Therefore, we can find $F(i)$ by finding the constant term of $F(x)/x^i$.

which is the expected result.

The two-dimensional(pile) case is more difficult, since we will not be able to do simple long division at the end to get t

Let $f(i,j) = f_{M,N}(i,j)$ be the expected duration of the game when the gambler has i dollars and j shekels. Then

with

Let

Then,