The Sagan-Savage Lucas-Catalan Polynomials Have Positive Coefficients

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In the last few minutes of Bruce Sagan's wonderful talk[1] about his joint work with Carla Savage[2] (whose notation I shall use), he mentioned that they can prove that $\frac{1}{\{n+1\}}\left\{ {2n \atop n} \right\}$ are polynomials with integer coefficients, and *conjectured* that they are in fact polynomials with *positive* coefficients.

But this follows immediately from the identity $\frac{1}{\{n+1\}}\left\{ {2n \atop n} \right\} = \left\{ {2n-1 \atop n-1} \right\} + t \left\{ {2n-1 \atop n-2} \right\}$ that after routine cancellations is equivalent to $\{2n\} = \{n+1\}\{n\} + t\{n-1\}\{n\}$, that is the case $m = n$ of Lemma 2.1 of [2]. $\square$

References

[1] Bruce Sagan, *Combinatorial Interpretations of Binomial Coefficient Analogues Related to Lucas Sequences*, talk at the Rutgers University Experimental Mathematics seminar on Dec. 9, 2010, videotaped by Edinah Gnang. http://www.youtube.com/watch?v=Fdn890jg2U0 .

[2] Bruce Sagan and Carla Savage, *Combinatorial Interpretations of Binomial Coefficient Analogues Related to Lucas Sequences*, Integers **10** (2010), 697-703, A52. http://arxiv.org/abs/0911.3159 .

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`http://www.math.rutgers.edu/~zeilberg/ekhad.html` . Jan. 17, 2011. Exclusively published in the Personal Journal of Shalosh B. Ekhad and Doron Zeilberger .