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Dear Committee,

This is a letter of recommendation on behalf of Tewodros Amdeberhan, who is applying for membership at IAS for AY 2002-2003. Tewodros earned his Ph.D. in 1997, under my direction.

Tewodros's thesis was a very important contribution to *algorithmic proof theory of identities*, with elegant and significant applications to *number theory* (Apéry-like irrationality proofs), and *determinant evaluations*. He has the very rare gift of being equally strong both in theory and computation. He is the master of the rigorous proof on the one hand, and at the same time he is a true computer whiz, who can write long Maple code in order to solve a problem.

Tewodros's thesis consists of three loosely related parts. The first part is in *convergence acceleration series*. Using WZ-cohomology, Tewodros found amazingly fast converging series for $\zeta(2)$ and $\zeta(3)$, much faster than Apéry's celebrated ones. One of his formulas:

$$\zeta(3) = \sum_{n=0}^{\infty} (-1)^n \frac{n!^{10}(205n^2 + 250n + 77)}{64(2n+1)!^5}.$$

was recently used by Simon Plouffe and George Fee to compute $\zeta(3)$ to more than half a million digit, a world record! See the Web page on mathematical constants at

`http://www.mathsoft.com`. This is also mentioned in the famous "Eric's Mathworld" website: `http://www.mathworld.com`.

The second, perhaps deepest, part of Amdeberhan's thesis is the irrationality proofs of the $q-$analogs of $\ln(2)$ and the Harmonic series (which converges when $q > 1$). These are

$$h_q(1) := \sum_{k=1}^{\infty} \frac{1}{(q^k - 1)}$$

and

$$Ln_q(2) := \sum_{n=1}^{\infty} \frac{(-1)^n}{q^n - 1}.$$

respectively. The special case $q = 2$ was done by Paul Erdös in 1947, and the general case was done recently by Peter Borwein. However, Amderberhan's methods give the best irrationality measure to date. More importantly, there is a good chance that they would generalize to irrationality proofs of the $q-$analogs of $\zeta(2)$ and $\zeta(3)$.

The third part is just as impressive. Tewodros proved a six-year-old conjecture of Greg Kuperberg and Jim Propp. In fact he did much more. He found a q-analog and a far-reaching generalization. The importance of exact evaluations of determinants is now rapidly increasing. I am sure that Tewodros has a very good chance to employ similar methods to prove a conjectured determinant evaluation of Bombieri, van der Poorten, and Hunt, that has far-reaching number-theoretic consequences.

In addition to Tewodros's mastery of Maple, he is very computer-literate. You are welcome to look him up in his own Home Page: `http://www.math.temple.edu/ ~ tewodros`.

Since his Ph.D., in spite of a very heavy teaching load, Tewodros continued to do excellent reseach branching out into differential equations.

Tewodros is also a gifted and concerned teacher, a very kind and responsible human being, who would make a great colleague. He is very generous with his time, and is always helping other students with their computer and mathematics problems.

To sum up, Tewodros is a very talented and promising mathematician, with equal strengths in theory, applications, and computations. He is also a very good teacher and a very fine human being.

Sincerely,

Doron Zeilberger
Board of Govs. Professor