Comment on 'Automatic Central Limit Theorems ...'

Herb Wilf

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1 Better recurrence

Recall that some years ago you were overly generous and credited Nijenhuis and myself with inventing the algorithm for raising a power series to the nth power, that appears in our book *Combinatorial Algorithms.* We pointed out that it was known long before us, and Don Knuth traced it back to the Neanderthals. Anyway that same algorithm gives a somewhat simpler recurrence for your $f_r(n)$'s in this paper.

In fact, if

$$\left(\sum_{r\geq 0}\frac{F_r}{r!}z^r\right)^n=\sum_{r\geq 0}\frac{f_r(n)}{r!}z^r,$$

then by logarithmic differentiation and matching coefficients one finds that

$$f_r(n)=\sum_{0\leq j<r}\left(n\binom{r-1}{j}-\binom{r-1}{j+1}\right)F_{j+1}f_{r-1-j}(n),$$

with $f_0=1$, $f_1=0$, which avoids the extra step of indefinite summation that's in your present recurrence.