ON AN IDENTITY OF DAUBECHIES

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Tossing a coin (whose $Pr(head) = p$) until reaching n heads or n tails and equating the probability,1, of finishing with the sum of the probabilities of all the possible final outcomes leads to

$$\sum_{i=0}^{n-1} \binom{n+i-1}{i} p^n (1-p)^i + \sum_{i=0}^{n-1} \binom{n+i-1}{i} p^i (1-p)^n = 1 \quad ,$$

which was proved in [1](pp. 167-171) and [2] using Bezout's theorem and induction respectively. Rolling a k-faced die instead leads to the multivariate generalization

$$\sum_{i=1}^{k} \sum_{\substack{0 \le \alpha_j \le n-1 \\ j \ne i}} \frac{(\alpha_1 + \ldots + \alpha_{i-1} + (n-1) + \alpha_{i+1} + \ldots + \alpha_k)!}{\alpha_1! \ldots \alpha_{i-1}!\,(n-1)!\,\alpha_{i+1}! \ldots \alpha_k!} p_1^{\alpha_1} \ldots p_{i-1}^{\alpha_{i-1}}\, p_i^n\, p_{i+1}^{\alpha_{i+1}} \ldots p_k^{\alpha_k} = 1 \quad ,$$

provided $p_1 + \ldots + p_k = 1$.

References

1. Daubechies, Ingrid, "*Ten Lectures on Wavelets*", SIAM, Philadelphia, 1992.
2.________, *Orthogonal bases of compactly supported wavelets*, Comm. Pure Appl. Math., **41**(1988), 909-996.