Multi-Gender Talmudic Family Planning

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Everybody knows that the dice are loaded - Leonard Cohen

Abstract: We harness the great power of symbolic computation and experimental mathematics to investigate the statistics of the number of (loaded) dice-rolls it takes until every face appears at least a certain desired, pre-assigned, number of times. We also investigate much more general scenarios.

Preface:

In a recent fascinating article that appeared in the Mathematical Intelligencer [BFV], Simon Blatt, Uta Freiberg, and Vladimir Shikhman investigated the mathematics of Talmudic Family Planning, where a couple has a predecided goal to "Go forth and multiply" until they have begotten n boys and k girls, for some pre-decided n and k. They stop as soon as they have reached *both* goals. They assumed that the probability of having a boy, p, is always the same, and the gender of each birth is independent of the others. They were wondering about the expected family size, that they called $F(n, k, p)$ in terms of n, k, and p, and proved the elegant *constant gender rule*, that asserts that if $B(n, k, p)$ and $G(n, k, p)$ are the expected number of boys and expected number of girls, respectively (of course $F(n, k, p) = B(n, k, p) + G(n, k, p)$) then

$$\frac{B(n,k,p)}{G(n,k,p)} = \frac{p}{1 = p} \quad .$$

Note that mathematically this is equivalent to tossing a coin whose probability of Heads is p (and hence the probability of Tails is $(1 - p)$) and stopping as soon as you have encountered at least n Heads and at least k Tails. The minimum number of coin-tosses is $n + k$, but the maximuma number is unbounded, after all you can have more than googolplex number of Heads before you get the first Tails.

In this paper we will generalize it to an **arbitrary** die, with k faces, let's call them $1, 2, \ldots, k$, where the probability of landing on face i is p_i, and of course

$$\sum_{i=1}^{k} p_i = 1 \quad .$$

While it is a bit silly, in homage to the article [BFV], we will keep the family planning interpretation where instead of a k-faced loaded die we have k genders each with its probability, and you have speficifed goals of achieving at least G_i babies of gedner g_i. We also have to stipulate, for this silly interpretation to make sense, how babies are born. Can any two members of different genders make a baby? But then their babies do not have to be from either the parents' genders. Perhaps that best is to decide that you need k individuals, one from each of the k gender to have *group sex* in order to begat a newborn.

References

[AMMZ] Noga Alon, Yaakov Malinovsky, Lucy Martinez, and Doron Zeilberger `Hitting k primes by dice rolls`, Electronic Journal of Combinatorics, to apper.

[BFV] Simon Blatt, Uta Freiberg, and Vladimir Shikhman, *The mathematics of family planning in the Talmud*, Math. Intelligencer **47(2)** (Spring 2025), 120-125.

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