with the property that no open connected subset of Y contains a cut point. Let A be a subset of X consisting of isolated points. Assume that the map $f\colon X \to Y$ *is continuous, and that* $f \upharpoonright X - A$ *is an open map.*

Then f is an open map.

Proof. It will suffice to show that f is open at x for each $x \in A$. Assume that there is an $x_0 \in A$ such that f is not open at x_0. Then there are open sets N and V, with $\overline{N}$ compact, such that $x_0 \in N \subset \overline{N} \subset V$, and $f(x_0) \notin f(V)^0$. Since the points in A are isolated, we may assume that $V \cap A = \{x_0\}$. Then since $f \upharpoonright X - A$ is an open map, $f(V - \{x_0\})$ is an open subset of $f(V)$, so $f(x_0) \notin f(V - \{x_0\})$.

Since $\overline{N} - \{x_0\} \subset V - \{x_0\}$, $f(x_0) \notin f(\overline{N} - \{x_0\})$, and $f(N - \{x_0\})$ is open. Since $f \upharpoonright \overline{N}$ is continuous and $\{x_0\}$ is not an open subset of N, $f(x_0) \in \overline{f(N - \{x_0\})}$. On the other hand, $f(\overline{N})$ is closed, so

$$\overline{f(N - \{x_0\})} \subset f(N - \{x_0\}) \cup f(\overline{N} - N) \cup \{f(x_0)\},$$

and since $f(\overline{N} - N)$ is closed and does not contain $f(x_0)$, it follows that $f(x_0)$ is an isolated point in the boundary of the open set $f(N - \{x_0\})$. We have $f(x_0) \notin \overline{f(N - \{x_0\})}^0$, because $\overline{f(N - \{x_0\})} \subset f(\overline{N}) \subset f(V)$, and $f(x_0) \notin f(V)^0$. Therefore, (ii) in Lemma 1 is false for Y, so (i) must also be false, contradicting our hypothesis. This concludes the proof of Theorem 1.

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On a Conjecture of R. J. Simpson About Exact Covering Congruences

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The following is a counterexample[2] to Simpson's conjecture [**2**]: $D = \{6, 15, 35, 14, 210 \text{ (140 times)}\}$. It was concocted using the elegant and powerful approach of [**1**].

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[1] Supported in part by NSF grant DMS 8800663.

[2] Another counterexample was found later, and independently, by John Beebee.