# EXPERIMENTS WITH A POSITIVITY PRESERVING OPERATOR

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ABSTRACT. We consider some multivariate rational functions which have (or are conjectured to have) only positive coefficients in their series expansion. We consider an operator that preserves positivity of series coefficients, and apply the inverse of this operator to the rational functions. We obtain new rational functions which seem to have only positive coefficients, whose positivity would imply positivity of the original series, and which, in a certain sense, cannot be improved any further.

## 1. INTRODUCTION

Are all the coefficients in the multivariate series expansion about the origin of

$$\frac{1}{1-x-y-z-w+\frac{2}{3}(xy+xz+xw+yz+yw+zw)}$$

positive? Nobody knows. For a similar rational function in three variables, Szegö [7] has shown positivity of the series coefficients using involved arguments. His dissatisfaction with the discrepancy between the simplicity of the statement and the sophistication of the methods he used in his proof has motivated further research about positivity of the series coefficients of multivariate rational functions. For several rational functions, including Szegö's, there are now simple proofs for the positivity of their coefficients available. For others, including the one quoted above [1], the positivity of their coefficients are long-standing and still open conjectures.

In this paper, we consider the positivity problem in connection with the operator $T_p$ ($p \geq 0$) defined as follows:

$T_p$: