A Generalization of Odlyzko's Conjecture: The Coefficients of $(1-q)^j/((1-q^{2n})...(1-q^{2n+2j}))$ **Alternate in Sign**

Jane Friedman and Doron Zeilberger

Appeared in: Proc. Amer. Math. Soc. **118**, 1013 (1993).

Let N, a, b, i, j, r, n denote arbitrary non-negative integers, and let

$$(x)_N := \prod_{i=0}^{N-1}(1-q^i x) \ \ , G(a,b) := \frac{(q)_{a+b}}{(q)_a (q)_b} \, , \ A := \{f(q)\, ; \, (-1)^i f^{(i)}(0) \geq 0, 0 \leq i < \infty\}.$$

A. Odlyzko conjectured that $(1-q)^j/(q)_j \epsilon A$. This was proved in [1], where the more general fact that $(1-q)^j G(2j,r) \epsilon A$ was proved. Here we show that this last result also implies the statement of the title, that generalizes Odlyzko's conjecture (for even j) in a different direction. To wit: set $t = q^{2n}, N = 2j$, in the q-binomial theorem $1/(t)_{N+1} = \sum_{r>=0} G(N,r)t^r$, and then multiply by $(1-q)^j$. QED.

Reference

1. D.Stanton and D. Zeilberger, *The Odlyzko conjecture and O'Hara's unimodality proof*, Proc. Amer. Math. Soc. **107**(1989), 39-42.

Department of Mathematics, University of San Diego, Alcala Park, San Diego, CA 92210.

Department of Mathematics, Temple University, Philadelphia, PA19122.

1980 Mathematics Subject Classification (1985 Revision). Primary 05A17,10A45.

The second author was supported by NSF grant DMS8901690.