

Numerical and Symbolic Experimentis with the Harry Dym Equation

Shalosh B. EKHAD and Doron ZEILBERGER

In fond memory of Harry Dym (Jan. 26, 1938 - July 18, 2024)

Abstract: We use both symbolic and numeric computations to experiment with the Harry Dym equation, of central important in soliton theory.

Theorem: Let $a_0(t)$, $a_1(t)$, and $a_2(t)$ be *any* formal power series in t . Then the first 10 terms in the ‘series expansion’, in x , of the *general solution* $u(x, t)$, of the celebrated Harry Dym equation

$$u_t = u^3 u_{xxx}$$

in the ring of formal power series in (x, t) are

$$\begin{aligned} & a_0(t) + a_1(t)x + a_2(t)x^2 + \frac{\frac{d}{dt}a_0(t)}{6a_0(t)^3}x^3 + \frac{\left(\frac{d}{dt}a_1(t)\right)a_0(t) - 3a_1(t)\left(\frac{d}{dt}a_0(t)\right)}{24a_0(t)^4}x^4 \\ & - \frac{3\left(\frac{d}{dt}a_1(t)\right)a_1(t)a_0(t) - 6a_1(t)^2\left(\frac{d}{dt}a_0(t)\right) - \left(\frac{d}{dt}a_2(t)\right)a_0(t)^2 + 3a_2(t)a_0(t)\left(\frac{d}{dt}a_0(t)\right)}{60a_0(t)^5}x^5 \\ & + \frac{x^6}{720a_0(t)^7} \cdot \\ & (36\left(\frac{d}{dt}a_1(t)\right)a_1(t)^2a_0(t)^2 - 18\left(\frac{d}{dt}a_1(t)\right)a_0(t)^3a_2(t) - 60a_1(t)^3a_0(t)\left(\frac{d}{dt}a_0(t)\right) - 18a_1(t)a_0(t)^3\left(\frac{d}{dt}a_2(t)\right) \\ & + 72a_1(t)a_0(t)^2a_2(t)\left(\frac{d}{dt}a_0(t)\right) + \left(\frac{d^2}{dt^2}a_0(t)\right)a_0(t) - 6\left(\frac{d}{dt}a_0(t)\right)^2) \end{aligned}$$

References

Shalosh B. Ekhad and Doron Zeilberger, Department of Mathematics, Rutgers University (New Brunswick), Hill Center-Busch Campus, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019, USA.
Email: ShaloshBEkhad at gmail dot com , DoronZeil] at gmail dot com .

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