

My So far unrealized Dream of Having Mourad Number One

Doron ZEILBERGER

Abstract: Even though Mourad Ismail is only six years my elder, I always considered him as a mentor.

Way way back, in the early eighties, I came across Mourad's very interesting papers with Dick Askey, Thaana Rashed, and M.V. Tamahankar. In particular, in [GRZ], together with Joe Gillis and Bruce Reznick, we observed that the fact that the power-series expansion of, for $0 \leq \lambda \leq 1$,

$$(1 - (1 - \lambda)x - \lambda y - \lambda xz - (1 - \lambda)yz + xyz)^{-\beta} \quad ,$$

all have positive power series coefficients when $\beta = 1$ brilliantly proved in [IT] via MacMahon's Master Theorem (one of my favorite theorems!), in a purely elementary and combinatorial way, immediately implies Tom Koornwinder's result [K] that it does so for all $\beta \geq 1$. Koornwinder's proof was far from elementary and was rather complicated.

I wrote Mourad a *fan mail* letter, and expressed the wish to collaborate, so that I will have *collaboration distance one* with him. I also added that this may help bring peace between Israel and Egypt (at the time they were 'enemies').

He replied with a very nice letter, that he will be happy to.

When I was up for tenure, in 1986, at Drexel University, after being hired in 1983 as a tenure-track associate professor, he was one of the people asked to write letters. His letter was so **positive** urging them that I was more than ready also to be promoted to *full professor*.

While I don't know it for sure, he probably wrote me letters when I moved from Drexel to Temple in 1990.

While my dream to collaborate with Mourad never came to be, I lived it *vicariously*. My mentor Joe Gillis, together with at-the-time his grad student, T. Offer, wrote a beautiful paper [GIO]. So my collaboration distance with Mourad is **two** via numerous routes.

The paper [GIO] is about the asymptotics of the number of multi-set derangements, extending the work in [AIR]. It is asymptotic analysis at its best. To my great surprise, and disappointment, I could not find it in *MathSciNet*, someone messed up! There are probably numerous other gems that MathSciNet (formerly Mathematics Reviews) missed.

Finally, one of my **all time favorites** is Mourad's *A simple proof of Ramanujan's ${}_1\psi_1$ sum*, that was published in 1977 in the Proceedings of the AMS, and was immortalized in George Andrews' classic 'Ten Lectures on q -series" **CBMS** 1986 (based on Andrews' lectures in a 1985 conference co-organized by Mourad at Arizona State University).

Thanks, Mourad, for all that great math, and warm friendship and mentoring.

References

- [AsIs] R. Askey and M.E.H. Ismail, *Permutation problems and special functions*, Canad. J. Math. **28** (1976), 853-874.
- [AIK] R. Askey, M.E.H. Ismail, and T. Koornwinder, *Weighted permutation problems and Laguerre polynomials*, J. Comb. Theory (Ser. A.) **25** (1978), 277-287.
- [AIR] R. Askey, M.E.H. Ismail, and M.T. Rashed, *A derangement problem*. Technical report 1522, UW-Madison Mathematics Research Center, June 1975.
- [G] J. Gillis, *The statistics of derangement-a survey*, Journal of Statistical Physics **58** (1990), 575-578.
- [GIO] J. Gillis, M. E. H. Ismail, and T. Offer, *An Asymptotic Problem in Derangement Theory*, SIAM Journal on Mathematical Analysis **21** (1990), 262-269.
- [GRZ] J. Gillis, B. Reznick, and D. Zeilberger *On Elementary methods in positivity theory*, SIAM J. of Mathematical Analysis **14** (1983), 396-399.
- [IT] M.E.H. Ismail and M.V. Tamahankar, *A combinatorial approach to some positivity problems*, SIAM J. Math. Anal. **10** (1979), 478-485.
- [K] T. Koornwinder, *Positivity proofs for linearization and connection coefficients of orthogonal polynomials*, J. London Math. Soc. **2** (1978), 101-114.

Doron Zeilberger, Department of Mathematics, Rutgers University (New Brunswick), Hill Center-Busch Campus, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019, USA.
Email: DoronZeil at gmail dot com .

Sept. 16, 2026