Two One-Line Proofs of Heron's Formula that says that the Area-Squared of a Triangle whose Side-Lengths are a,b,c is (a+b+c)(a+b-c)(a+c-b)(b+c-a)/16

Shalosh B. EKHAD and Doron ZEILBERGER[1]

Maple Proof (by SBE): In a Maple session type:

```
evalb(expand(subs(c=1,b=sqrt(x^2+y^2),a=sqrt((x-1)^2+y^2),(a+b+c)*(a+b-c)*(a+c-b)*(b+c-a)/16-(y/2)^2))=0);
```

and get (after `0.` seconds!): `true`.

Clarification: Without loss of generality the vertices of the triangle are at $A(0,0)$, $B(1,0)$, and $C(x,y)$. So $c=1$, $b=\sqrt{x^2+y^2}$, $a=\sqrt{(x-1)^2+y^2}$, and the area is $1 \cdot y/2 = y/2$.

Human Proof (by DZ): The area-squared is obviously a *symmetric* and *homogeneous* polynomial of degree 4 in a,b,c, divisible by $(a+b-c)(a+c-b)(b+c-a)$, since degenarate triangles have zero area. Hence the area-squared divided by $(a+b-c)(a+c-b)(b+c-a)$ is a symmetric and homogeneous polynomial of degree 1 in a,b,c, and so is $(a+b+c)$ times some constant that must be $\frac{1}{16}$ by considering, say, the $90^o, 45^o, 45^o$ triangle.

Comments: **1.** These two proofs were inspired by the excellent MAA invited talk[D] by William Dunham, where I was astonished on how such great people as Hero, Newton, and Euler, and last-but-not least, the great inventor Bernard ("Barney") Oliver gave complicated proofs to this utter triviality. I am sure that all the human-generated mathematics published today, by great people such as Andrew Wiles and Yitang Zhang, would be considered utterly trivial in a hundred years. Our only hope at non-trivial mathematics is to use computers!

2. The above two proofs are much shorter (and nicer!) than all the proofs in [W].

References

[D] William Dunham, *Heron, Newton, Euler, and Barney*, MAA invited talk, JMM, Baltimore,Saturday January 18, 2014, 10:05 a.m.-10:55 a.m,
abstract: `http://jointmathematicsmeetings.org/amsmtgs/2160_abstracts/1096-a0-12.pdf`

[W] Wikipedia, *Heron's Formula*, `http://en.wikipedia.org/wiki/Heron`

[1] Department of Mathematics, Rutgers University (New Brunswick), Hill Center-Busch Campus, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019, USA. `zeilberg at math dot rutgers dot edu` , `http://www.math.rutgers.edu/~zeilberg/` . Jan. 20, 2014. Supported in part by the NSF.