

Investigating the Statistics of the Number of Dice-Rolls Until Reaching a Desired Goal

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Everybody knows that the dice are loaded - Leonard Cohen

Abstract: We harness the great power of symbolic computation and experimental mathematics to investigate the statistics of the number of (loaded) dice-rolls it takes until every face showed at least a certain desired number of times. We also investigate much more general scenarios.

Preface:

In a recent fascinating article that appeared in the Mathematical Intelligencer, Simon Blatt, Uta Freiberg, and Vladimir Shikhman investigated the mathematics of Talmudic Family Planning, where a couple has a predecided goal to “Go forth and multiply” until they have begotten n boys and k girls, for some pre-decided n and k . They stop as soon as they have reached *both* goals. They assumed that the probability of having a boy, p , is always the same, and the gender of each birth is independent of the others. They were wondering about the expected family size, that they called $F(n, k, p)$ in terms of n , k , and p , and proved the elegant *constant gender rule*, that asserts that if $B(n, k, p)$ and $G(n, k, p)$ are the expected number of boys and expected number of girls, respectively (of course $F(n, k, p) = B(n, k, p) + G(n, k, p)$) then

$$\frac{B(n, k, p)}{G(n, k, p)} = \frac{p}{1 - p} \quad .$$

Of course, mathematically this is equivalent to tossing a coin whose probability of Heads is p (and hence the probability of Tails is $(1 - p)$) and stopping as soon as you have encountered at least n Heads and at least k Tails. The minimum number of coin-tosses is $n + k$, but the total number is arbitrary.

In this paper we will generalize it to an **arbitrary** die, with k faces, let’s call them $1, 2, \dots, k$, where the probability of landing on face i is p_i , and of course

$$\sum_{i=1}^k p_i = 1 \quad .$$

References

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