Appendix to Cutting 4 by n grids into two congruent pieces

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This theorem also follows from the grammar, but was omitted from the paper.

Theorem 2: Let $f_a(x)$ be the coefficient of z^a in the Maclaurin expansion (with respect to the variable z) of the rational function

$$\frac{-x^3-x^2+x+1+\left(x^5+2x^3-x^2-x\right)z+\left(x^4-x^3+2x^2-1\right)z^2}{z^3+z^2-3z+1}$$

then the number of $4\times n$ good matrices whose first row starts with a consecutive zeroes (and hence ends with $n-a$ consecutive ones) is the coefficient of x^n in the Maclaurin expansion of the rational function

$$\frac{x^{2a}f_a(x)}{(x-1)\left(x^4+3x^2-1\right)} \quad .$$

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