

Comment on ‘Automatic Central Limit Theorems ...’

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1 Better recurrence

Recall that some years ago you were overly generous and credited Nijenhuis and myself with inventing the algorithm for raising a power series to the n th power, that appears in our book *Combinatorial Algorithms*. We pointed out that it was known long before us, and Don Knuth traced it back to the Neanderthals. Anyway that same algorithm gives a somewhat simpler recurrence for your $f_r(n)$ ’s in this paper.

In fact, if

$$\left(\sum_{r \geq 0} \frac{F_r}{r!} z^r \right)^n = \sum_{r \geq 0} \frac{f_r(n)}{r!} z^r,$$

then by logarithmic differentiation and matching coefficients one finds that

$$f_r(n) = \sum_{0 \leq j < r} \left(n \binom{r-1}{j} - \binom{r-1}{j+1} \right) F_{j+1} f_{r-1-j}(n),$$

with $f_0 = 1$, $f_1 = 0$, which avoids the extra step of indefinite summation that’s in your present recurrence.