New Sequence Problems and Solutions from 2022

Experimental Math Seminar, Rutgers, September 15 2022

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The Best Sequences of 2022

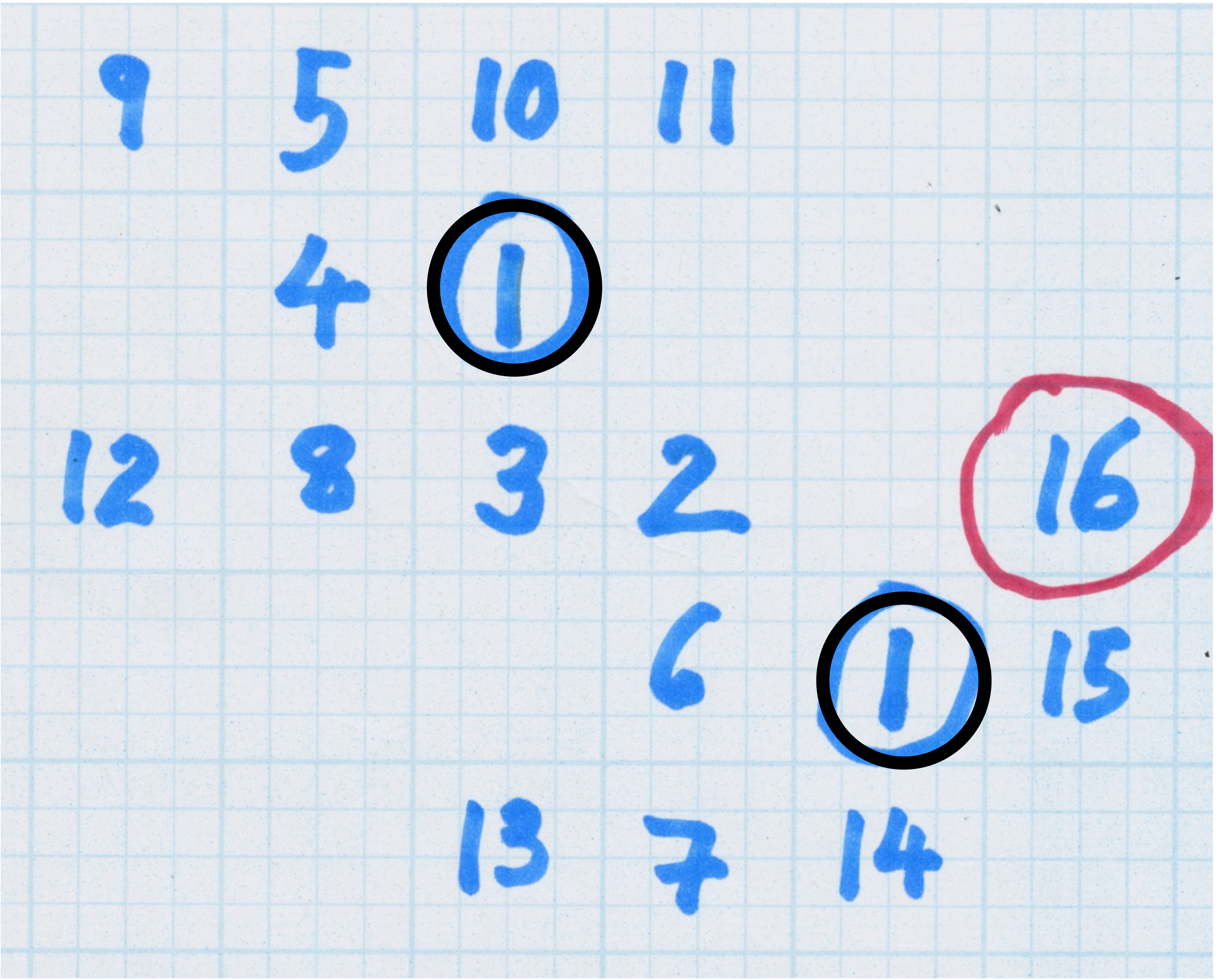
Outline

- Stepping Stones (updated)
- Scott Shannon’s Magic Carpet
- Augusto Santi’s Recurrence
- The Sisyphus Sequence
- Biggest number of 2022 ?
- Kaprekar’s Junction Numbers (with Max Alekseyev)
- The Binary Two-Up Sequence (with Rémy Sigrist et al.)
- Other topics, including most-wanted-formulas

New Results on the
Stepping Stones Puzzle

Thomas Ladouceur and Jeremy Rebenstock, October 2020

Start by placing n 1's on infinite square grid.
Then write 2, 3, 4, …, m subject to condition
that when you write k, the sum of its neighbors must equal k.
Maximize m.



a(2) = 16

A337663

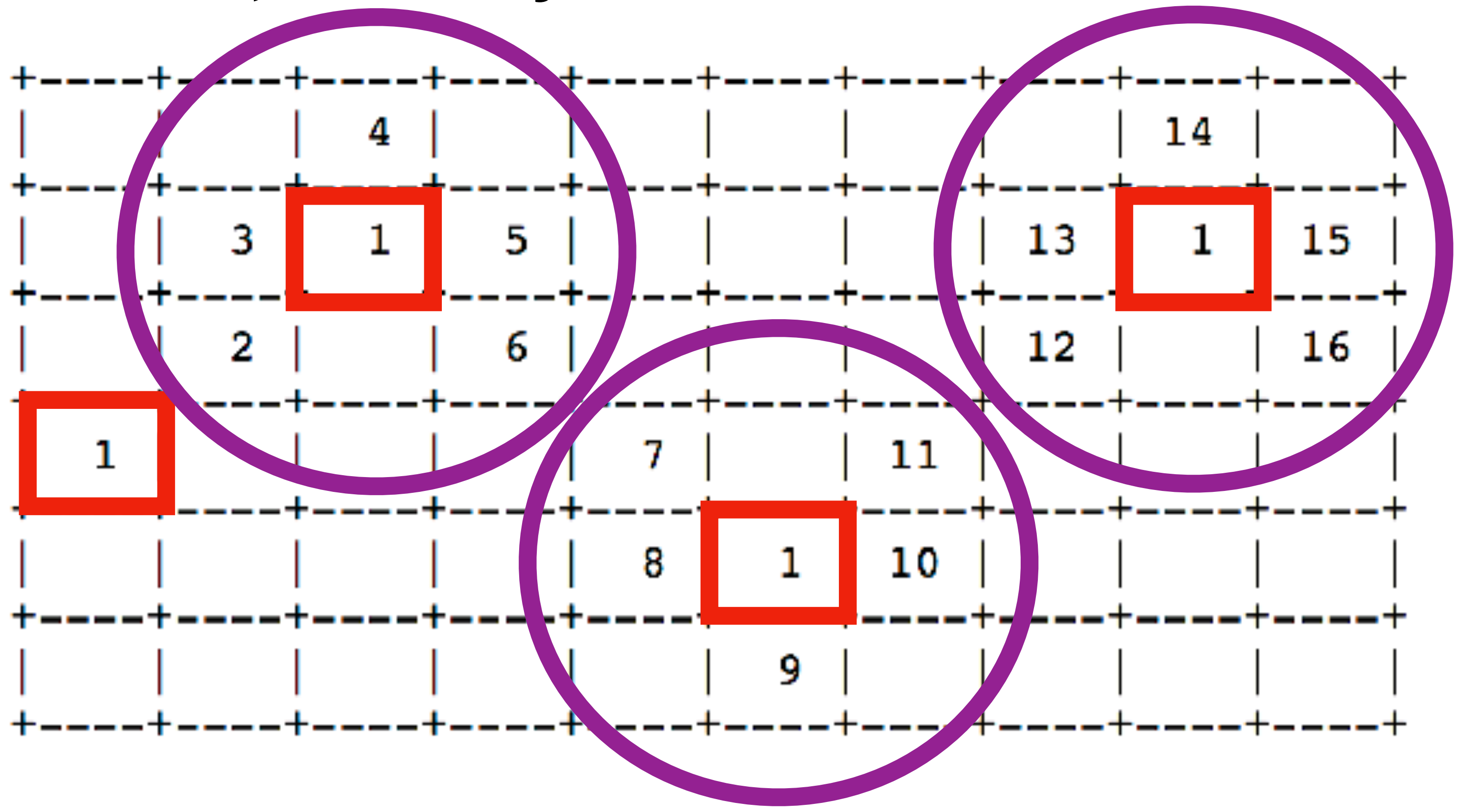
Stepping Stones Puzzle A337663 (cont.)

Video

Brady Haran and N. J. A. Sloane,
https://www.youtube.com/watch?v=m4Uth-EaTZ8,
Stones on an Infinite Chessboard, Numberphile video, January 2022.

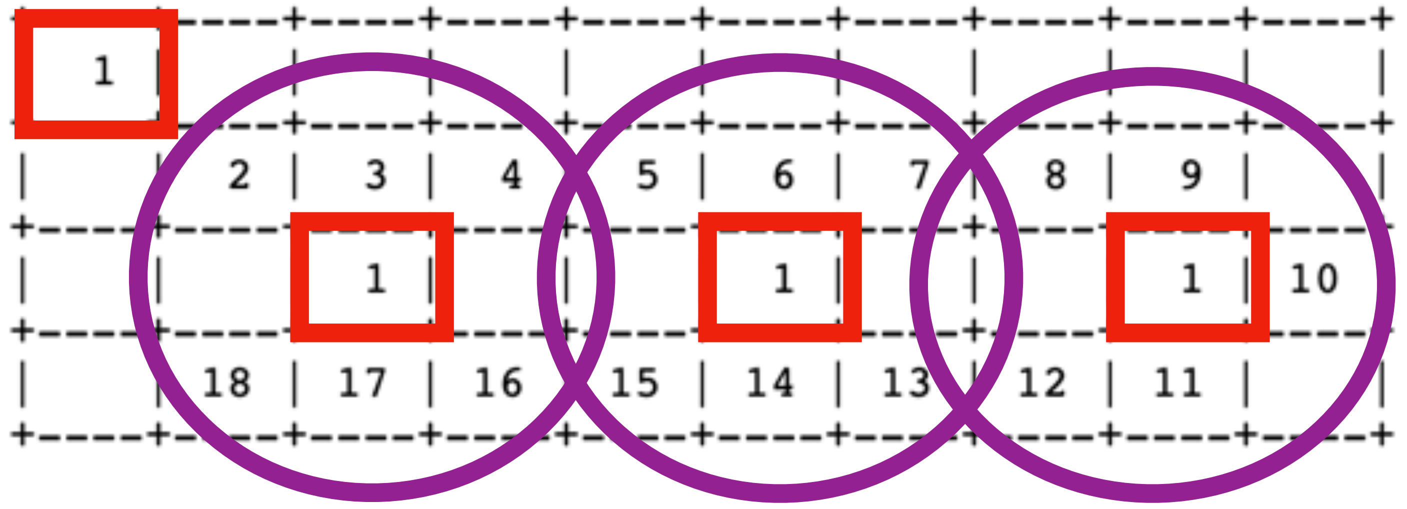
Old lower bound (Andrew Howroyd):

a(n) >= 5n-4



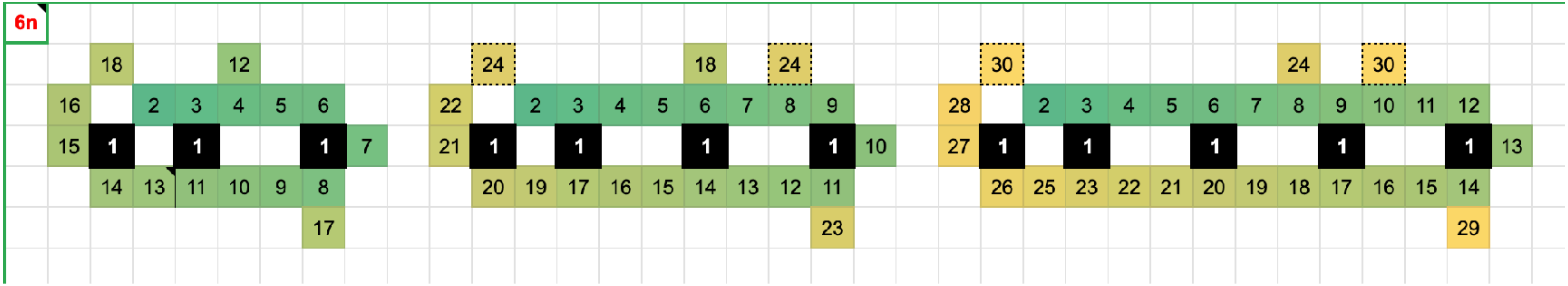
New lower bounds (Skylark Murphy-Davies):

a(n) >= 6n-6



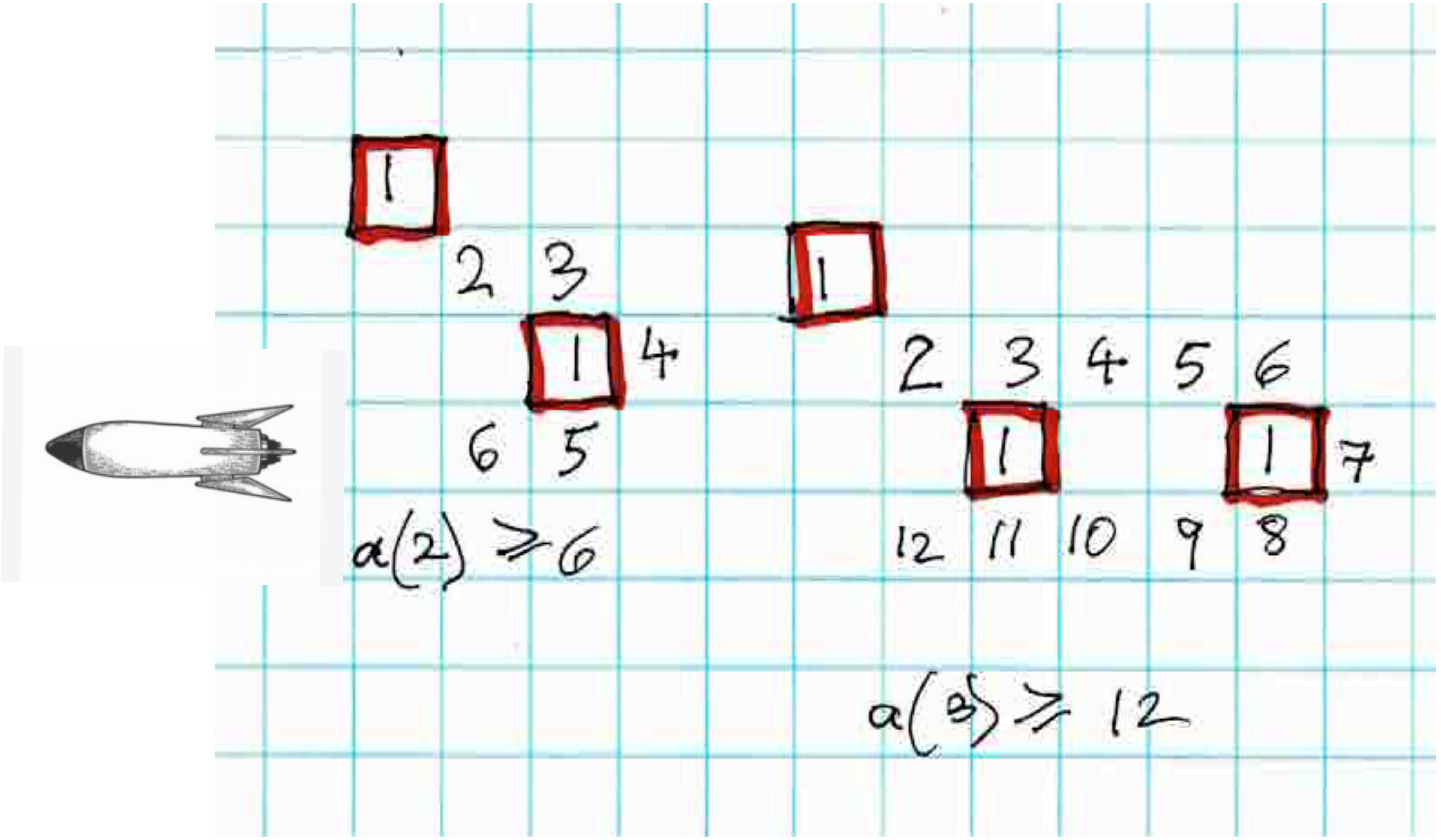
but even better:

a(n) >= 6n for n >= 3:



This isn’t rocket science.
But wait, maybe it is

Start by placing n 1’s on an infinite square grid.
Then write 2, 3, 4, …, m subject to condition
that when you write k, the sum of its eight neighbors must equal k.
And the path 2, 3, 4, …, m must be vertex-connected!
Maximize m.



Stepping Stones Puzzle A337663 (cont.)

Old upper bound (Robert Gerbicz): $a(n) < 714\,n$

Many new upper bounds.
Most recent (Jonathan F. Waldmann): $a(n) < 86\,n + 32$ (see link in A337663)

Exact values (no change):

a(1) = 1, a(2) = 16, a(3) = 28, a(4) = 38, a(5) = 49, a(6) = 60

But Al Zimmermann is running a programming contest
to improve the lower bounds (see http://azspcs.com/Contest/SteppingStones)
resulting in many improvements to the old bounds. In particular:

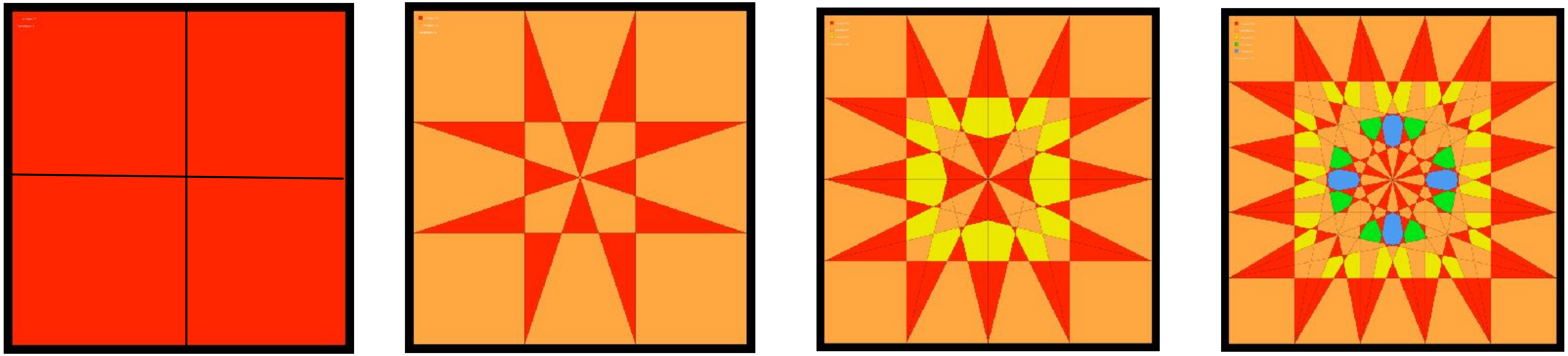
a(7) >= 71, a(8) >= 79, a(9) >= 89, a(10) >= 99, a(11) >= 109, a(12) >= 115.

The Best Picture of the Year Award goes to Scott Shannon for A355798

Scott Shannon’s Sequence A355798

Place n-1 points on each side of a square,
join each point to every point on the opposite side.
How many regions?

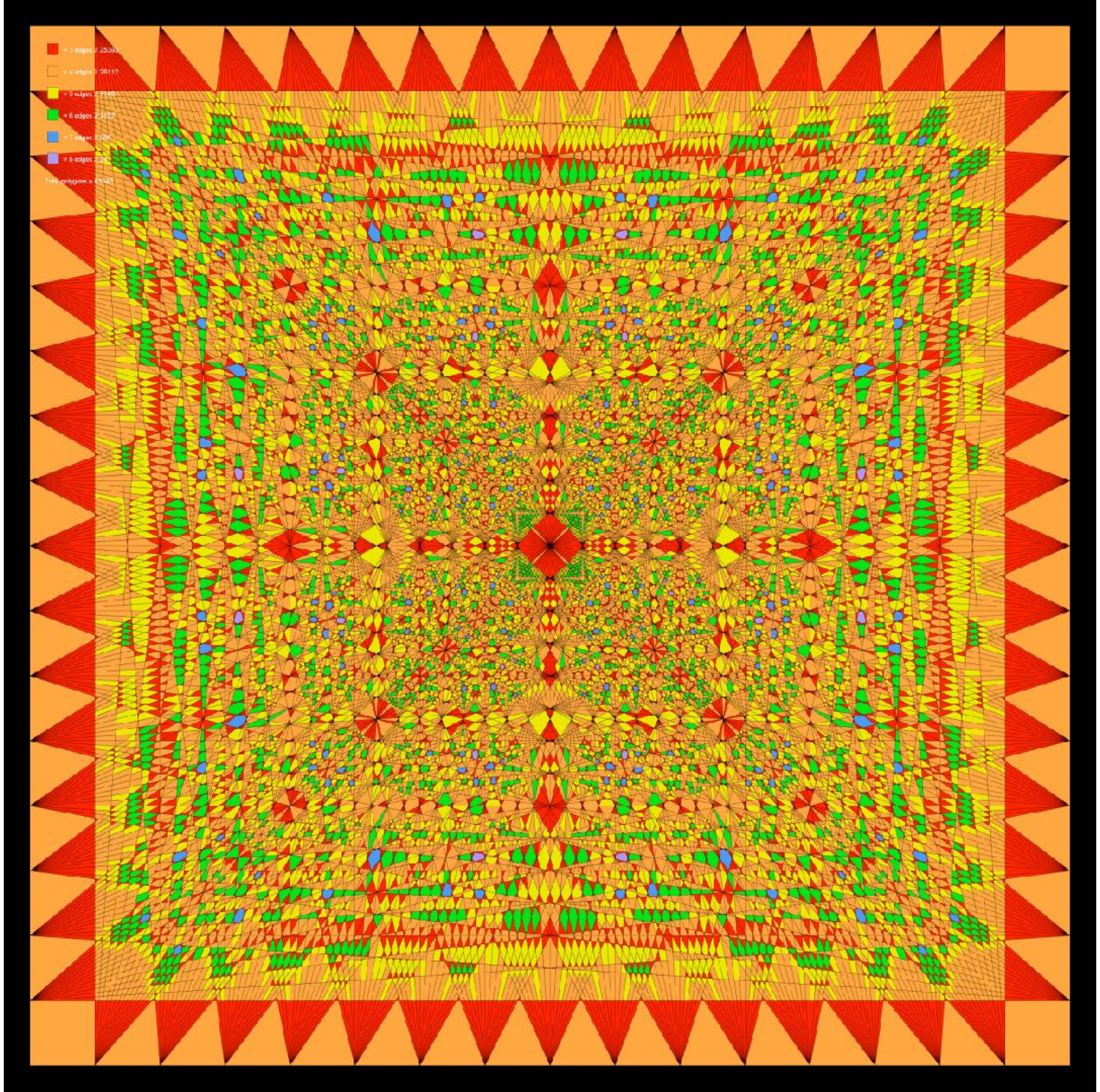
1, 4, 24, 104, 316, 712, 1588, 2816, 4940, 7672, …



Open Problem: Have 40 terms, need a formula

Also A355799 (vertices) and A355800 (edges)

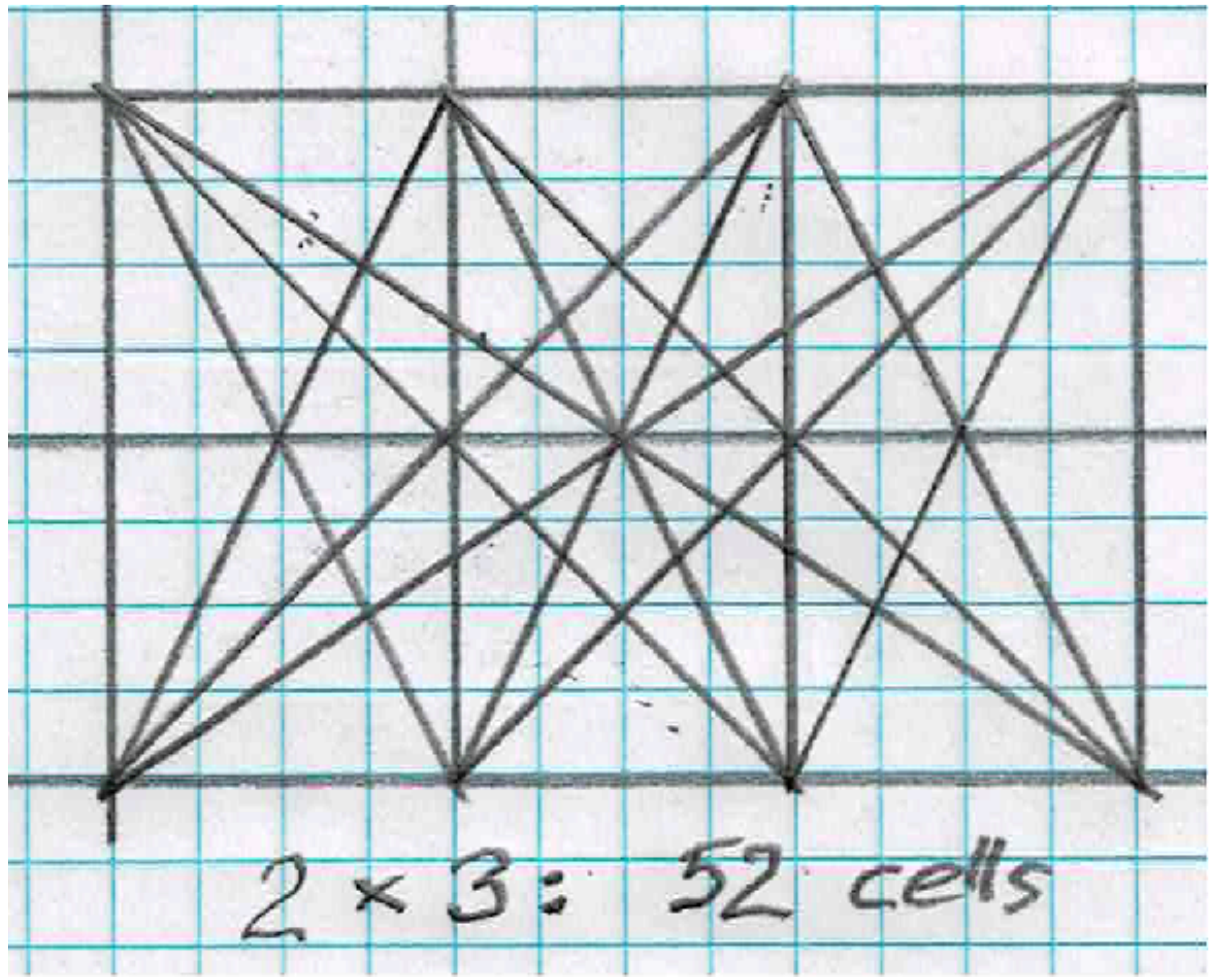
Scott Shannon's Magic Carpet



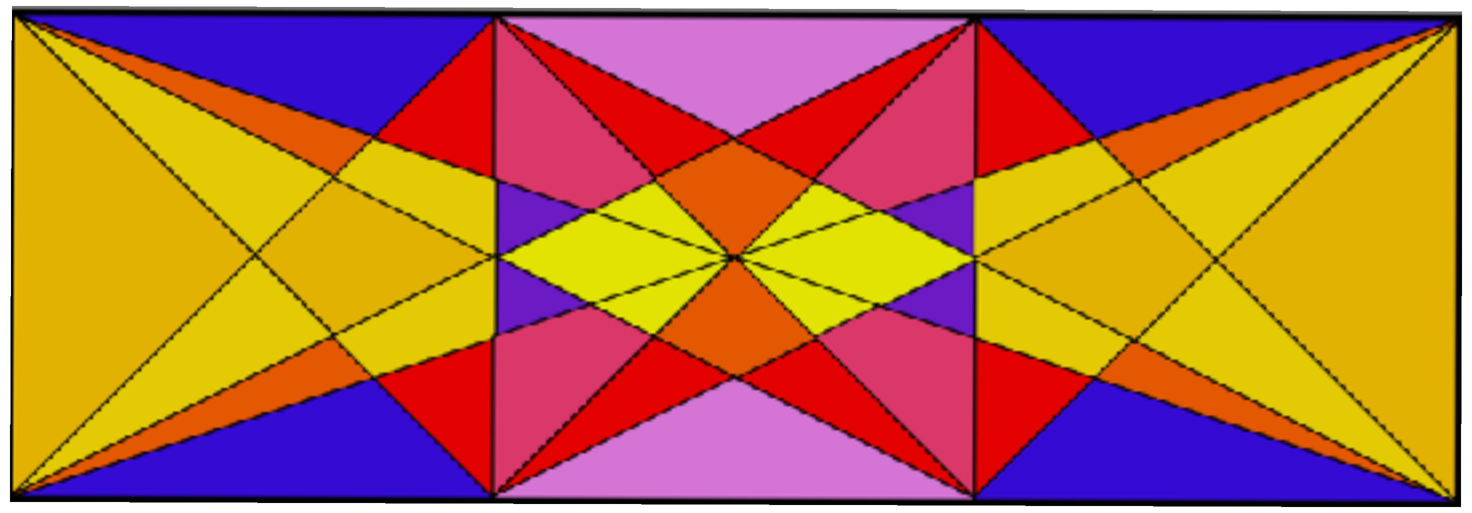
n=16
61408 regions


= 3 edges X 25392
= 4 edges X 26112
= 5 edges X 7160
= 6 edges X 2152
= 7 edges X 208
= 8 edges X 24
Total polygons = 61048

A355902



Scott Shannon: Remove the central line!



(A306302)

A SIMPLER VERSION: Two rows only

2 X n array of squares, connect all nodes on the long edges. How many cells?

Then divide by 2. The initial terms a(n) are:

0, 3, 10, 26, 56, 112, 196, 331, 522, 790, 1138, 1615, ...
Have 50 terms (A355902)

and we get a graph we analyzed in

Lars Blomberg, Scott R. Shannon and N. J. A. Sloane,
Graphical Enumeration and Stained Glass Windows, 1: Rectangular Grids,
(2020). Also arXiv:2009.07918.

(A355902 cont) Thanks to Max Alekseyev:

M. A. Alekseyev, M. Basova, and N. Yu. Zolotykh. On the minimal teaching sets of two-dimensional threshold functions, SIAM Journal on Discrete Mathematics 29:1 (2015), 157-165.

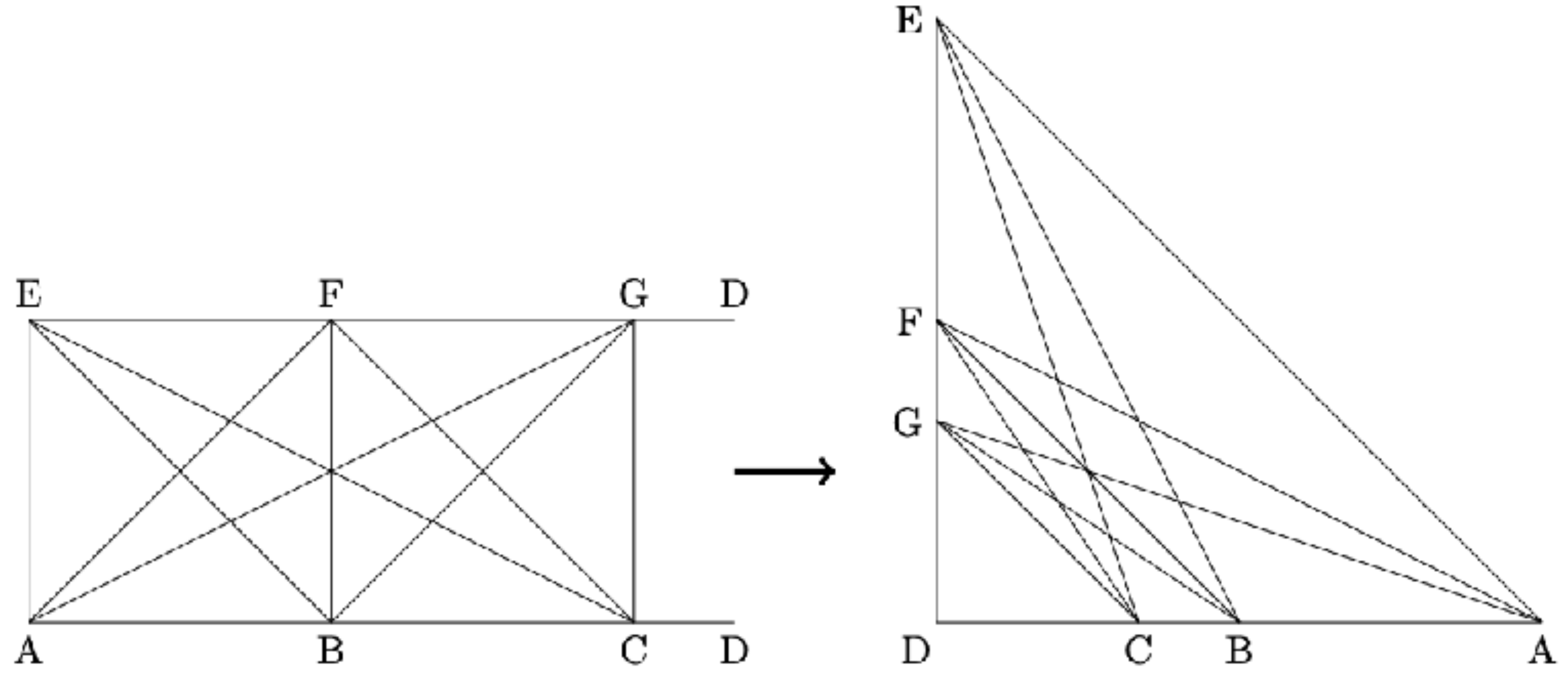


Figure 12: Illustrating the map (3.1) from $BC(1,2)$ to $IT(2)$.

Essentially the same sequence as A115004:

$$z(n) = \sum_{i,j = 1..n,\ \gcd(i,j) = 1} (n+1-i)(n+1-j)$$

The following eight sequences are all essentially the same. The simplest is the present sequence, **A115004**(n), which we denote by z(n). Then A088658(n) = 4*z(n-1); A114043(n) = 2*z(n-1)+2*n^2-2*n+1; A114146(n) = 2*A114043(n); A115005(n) = z(n-1)+n*(n-1); A141255(n) = 2*z(n-1)+2*n*(n-1); A290131(n) = z(n-1)+(n-1)^2; A306302(n) = z(n)+n^2+2*n.

Finally, a(n) = (z(n) + n^2 + 4n)/2.

But the big question remains: Find a formula for the Magic Carpet sequence

Augusto Santi’s Recurrence (A351871)

Reed Kelley's Sequence A214551

14th century Narayana cows sequence A930:

$$a(n) = a(n-1) + a(n-3)$$

1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, ...

Reed Kelley, 2012:

$$a(n) = \frac{a(n-1) + a(n-3)}{\gcd\{a(n-1), a(n-3)\}}$$

1, 1, 1, 2, 3, 4, 3, 2,
3, 2, 2, 5, 7, 9, 14, 3,...

(Have guesses, but nothing is proved.)

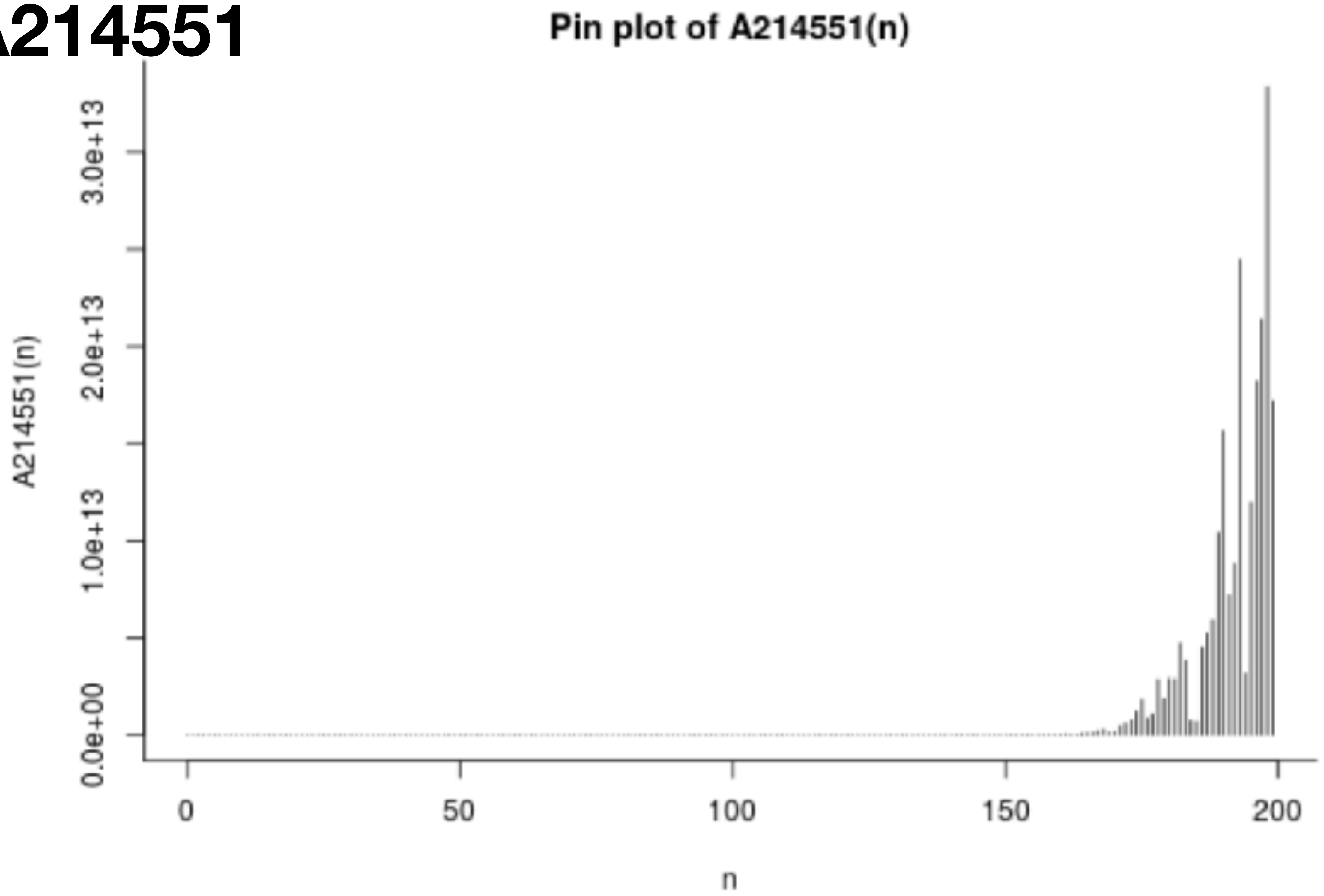
Reed Kelley's Sequence A214551

$$a(n) = \frac{a(n-1) + a(n-3)}{\gcd\{a(n-1), a(n-3)\}}$$

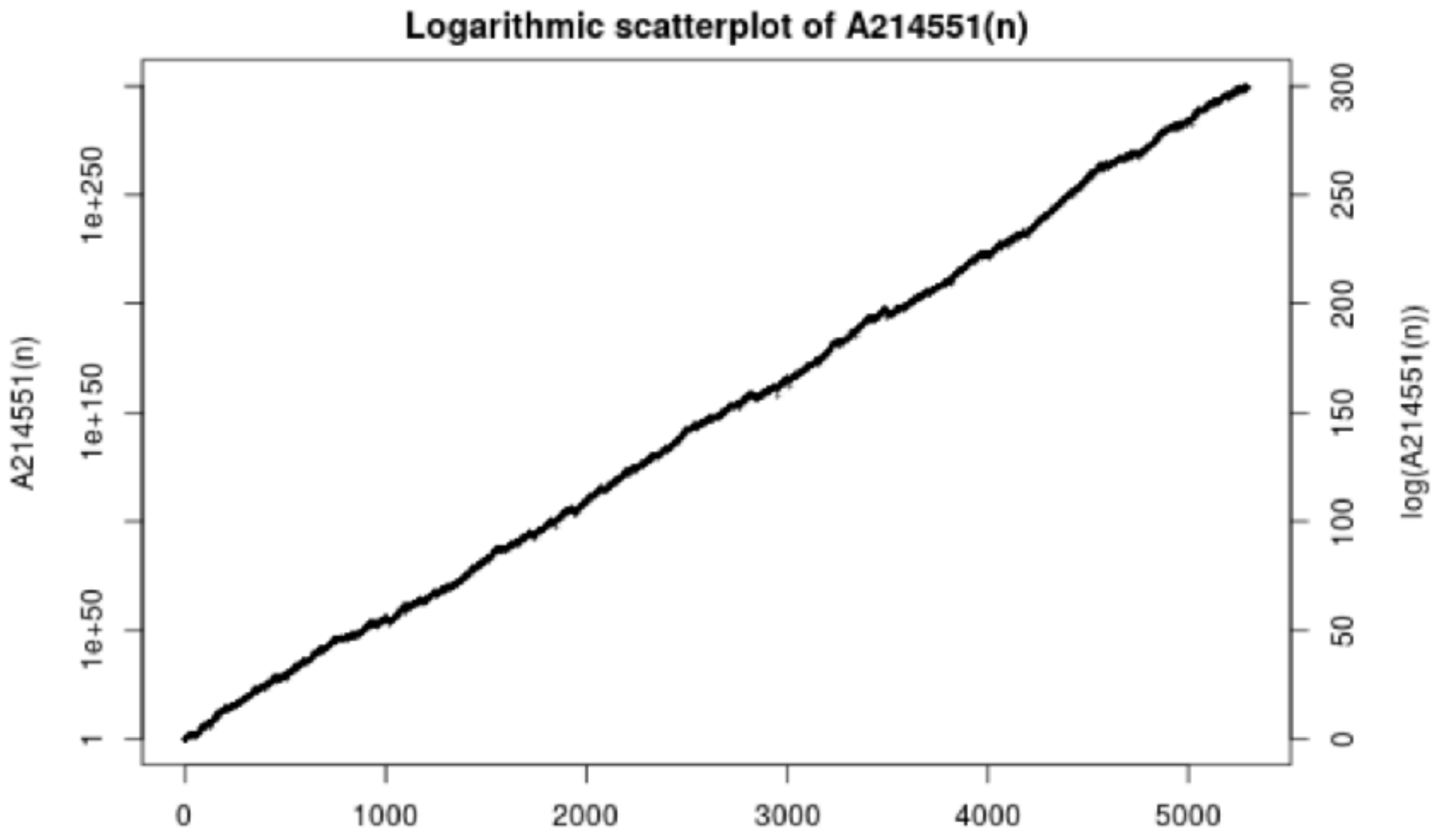
1, 1, 1, 2, 3, 4, 3, 2, 3, 2, 2, 5, 7, 9, 14, 3, 4, 9, 4, 2, 11, 15, 17, 28, 43, 60, 22, 65, 25, 47, 112, 137, 184, 37, 174, 179, 216, 65, 244, 115, 36, 70, 37, 73, 143, 180, 253, 36, **6**, 259, 295, 301, 80,

Open Problem: Show 8 and 10 never appear

Reed Kelley's Sequence A214551



Open Problem: Explain the wobbles



Augusto Santi's A351871

Feb. 2022

$(x, y) := \gcd(x, y)$ start 1, 2 $a_n = (a_{n-2}, a_{n-1}) + \frac{a_{n-2} + a_{n-1}}{(a_{n-2}, a_{n-1})}$

1, 2, 4, 5, 10, 8, 11, 20, 32, 17, 50, 68, 61, 130, ...

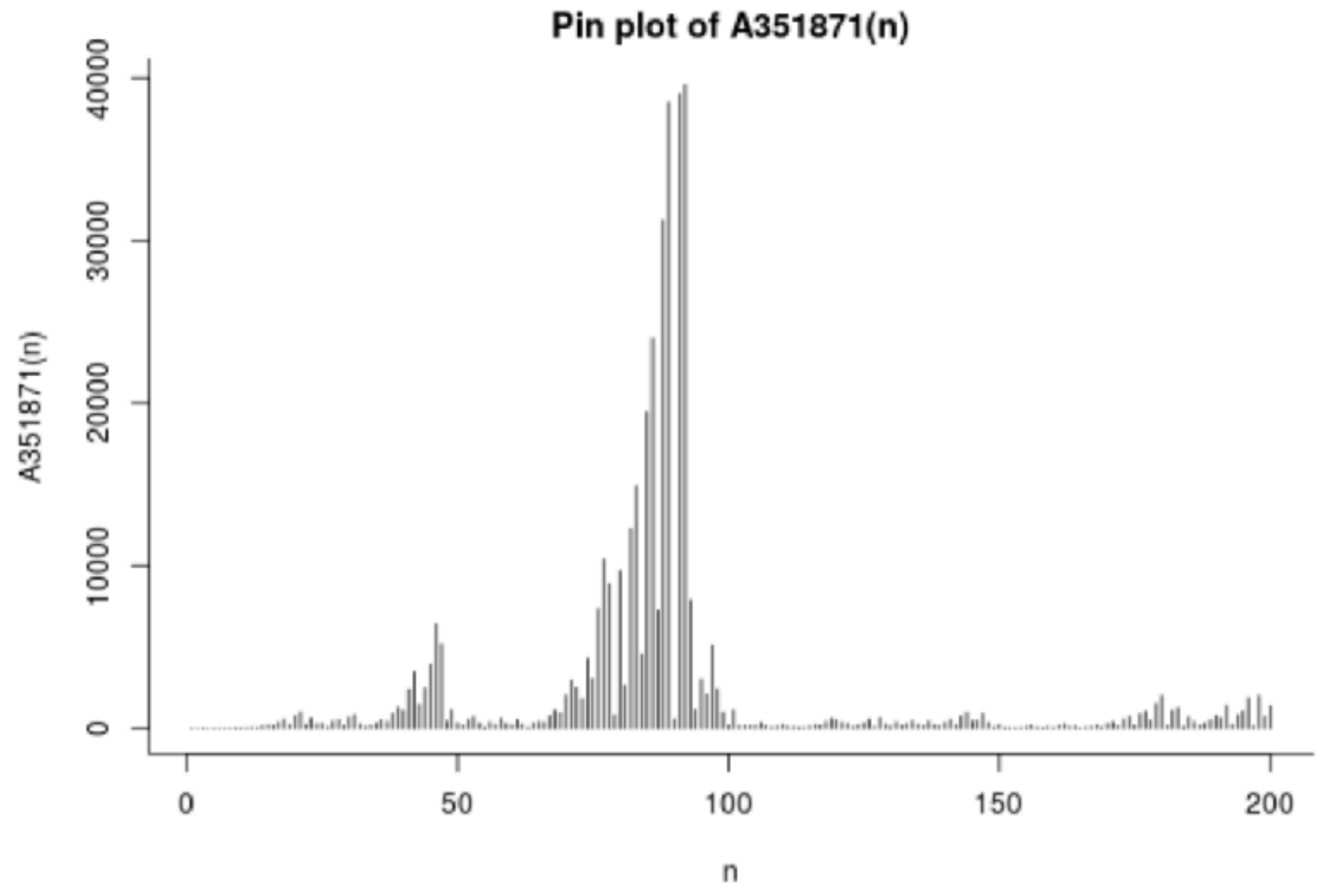
After 2006 steps, reaches 226942931276539547082O,
and repeats with period 2901

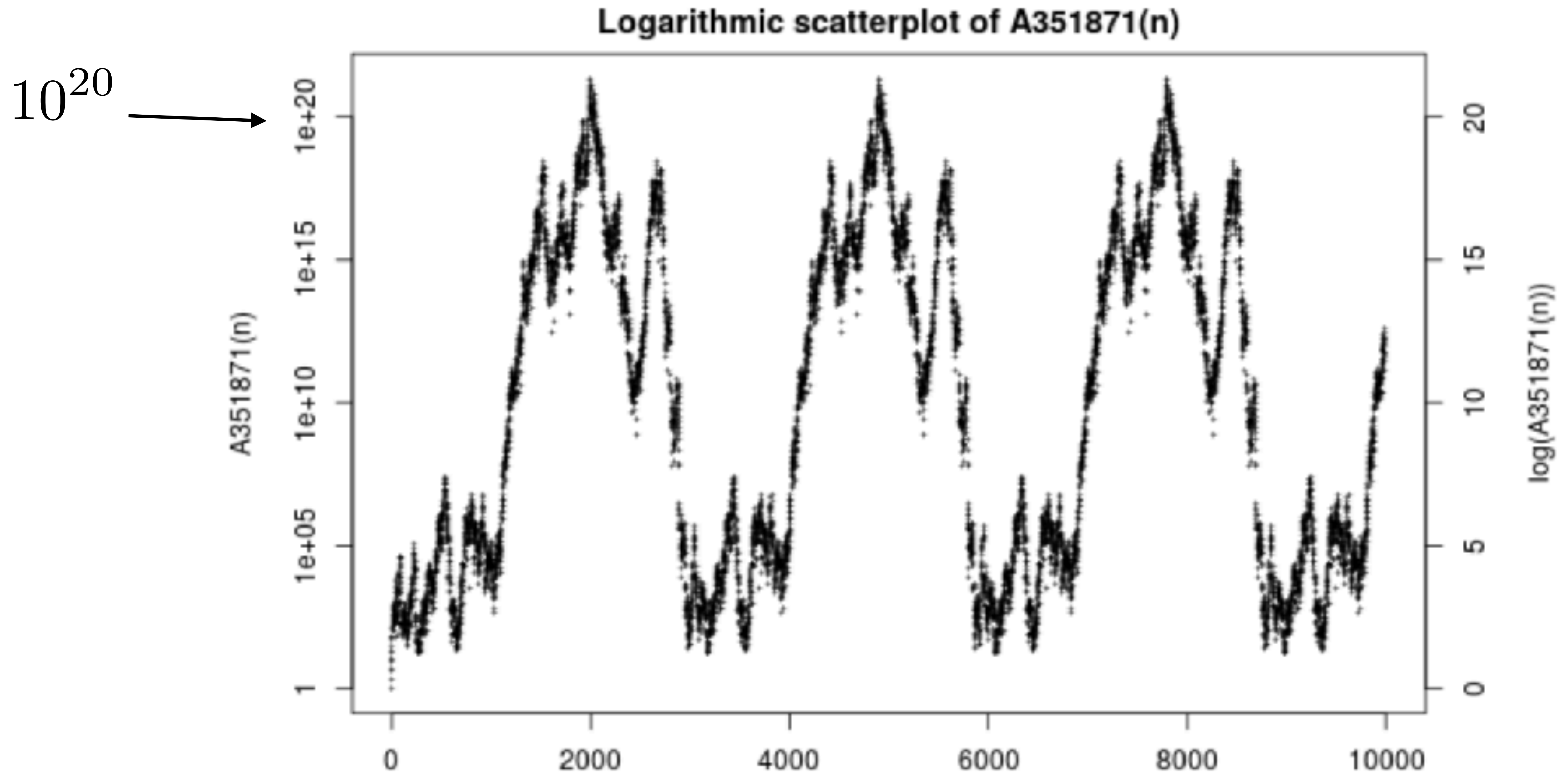
With different initial terms, it seems that either the sequence diverges or is periodic with period length 3, 9, 155, or 2901.

Open Problem: Are these the only possible periods?

(And what is the theory behind this sequence?)

Augusto Santi's A351871

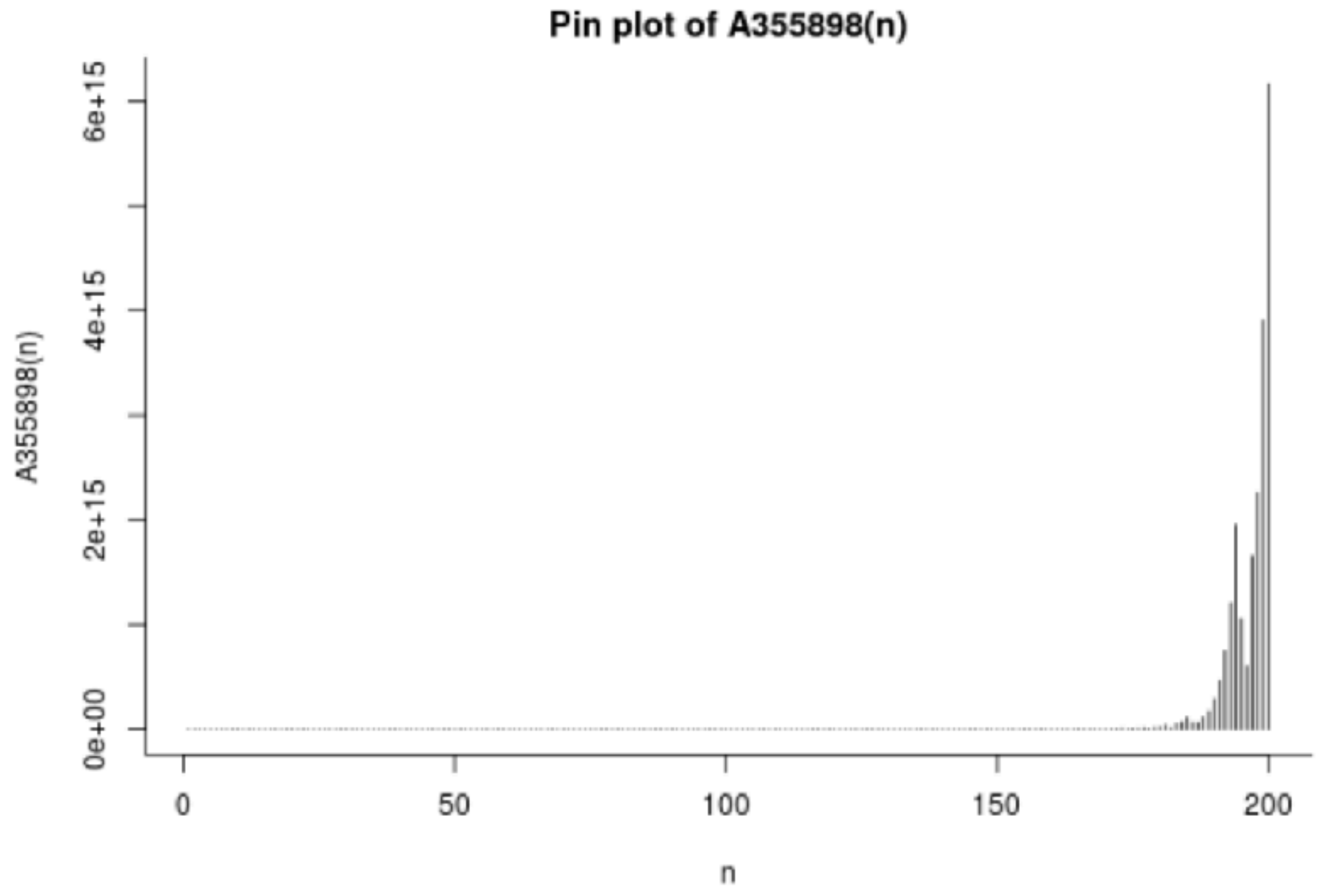


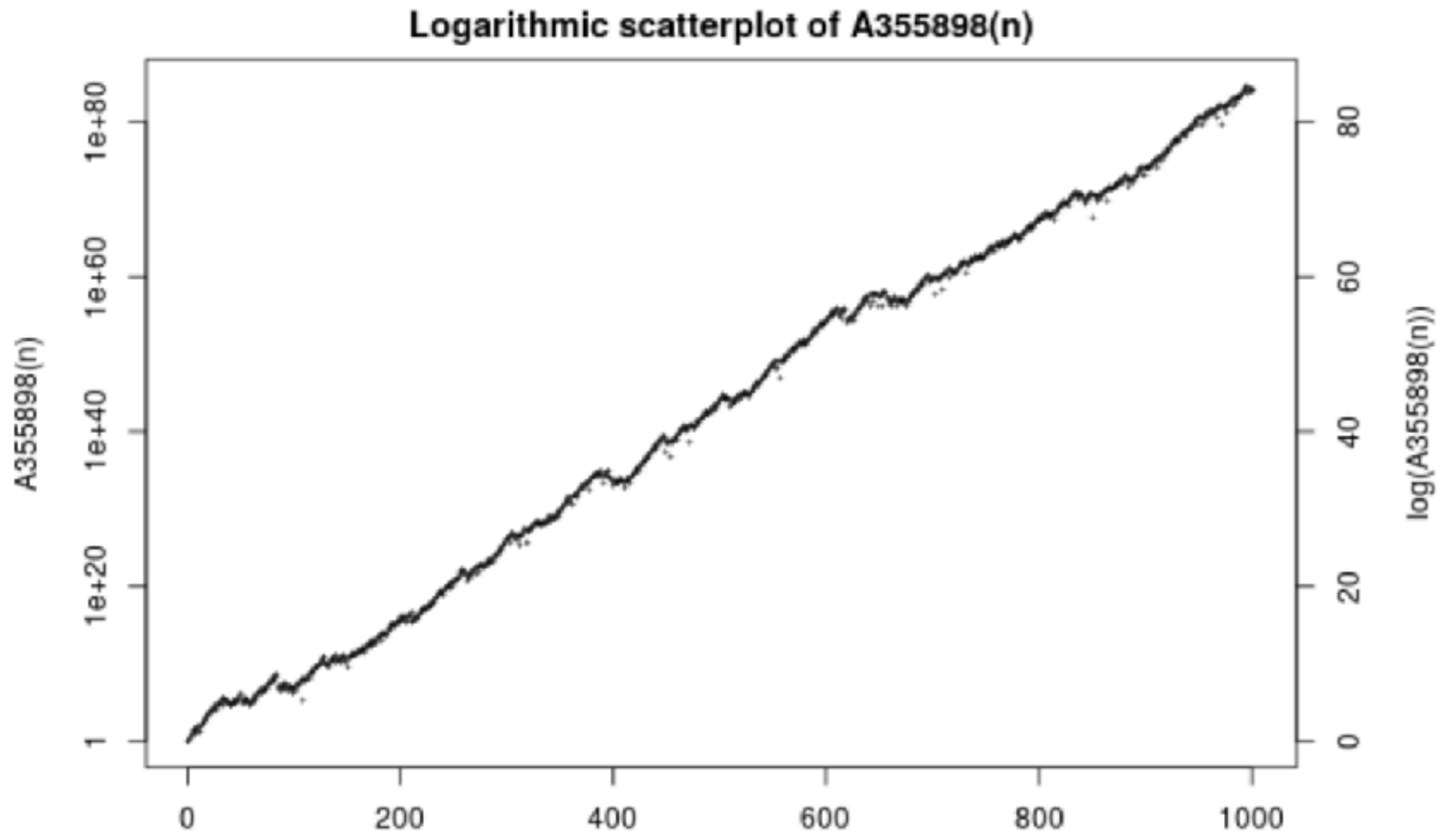


Augusto Santi's A351871

Open Problem: Show that with initial terms 1, 1 the sequence (A355898) diverges.

1, 1, 3, 5, 9, 15, 11, 27, 39, 25, 65, 23, 89, 113, 203, 317, 521, …





The Sisyphus Sequence

A350877

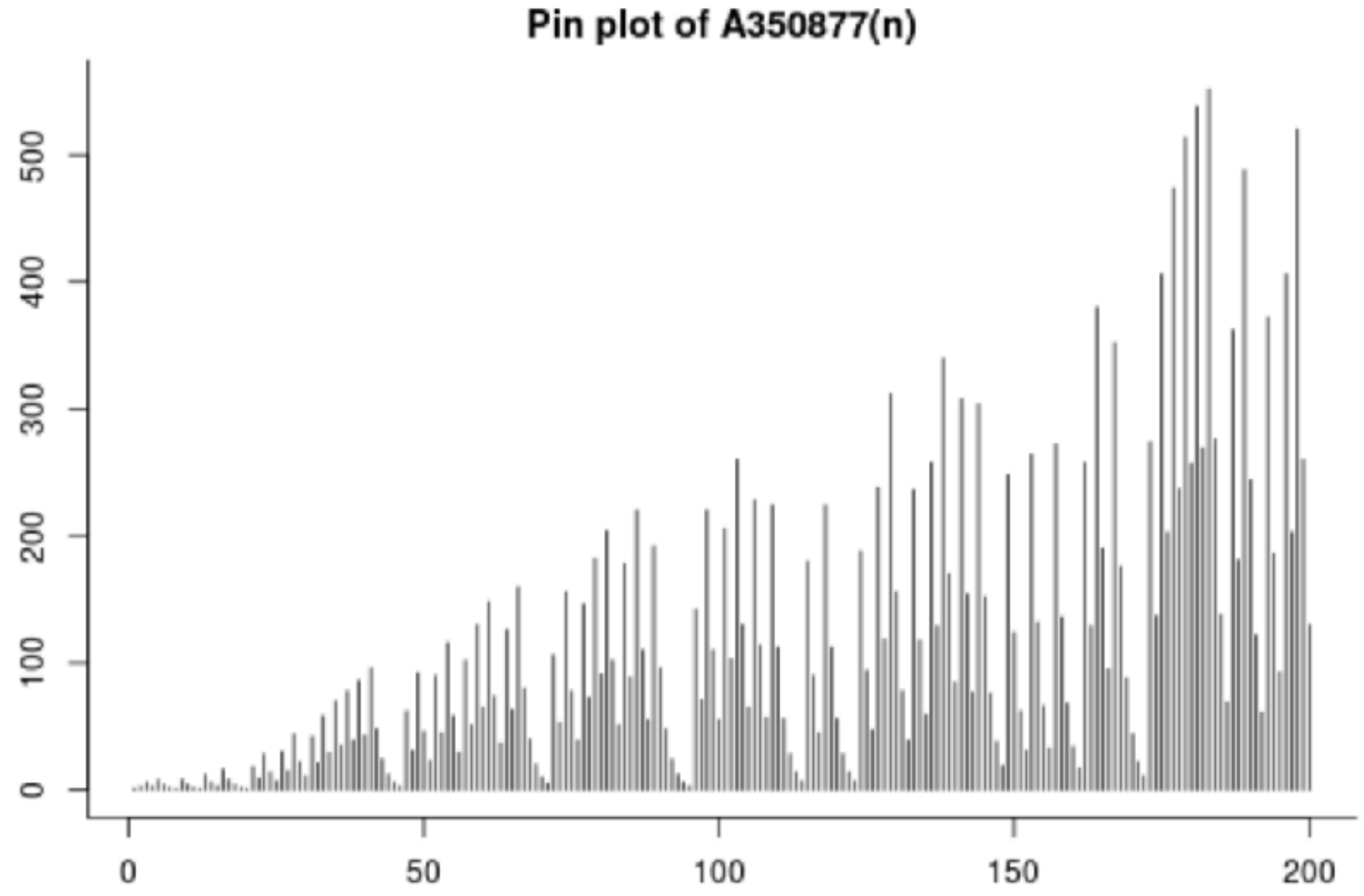
Éric Angelini and Carole Dubois, January 2022;

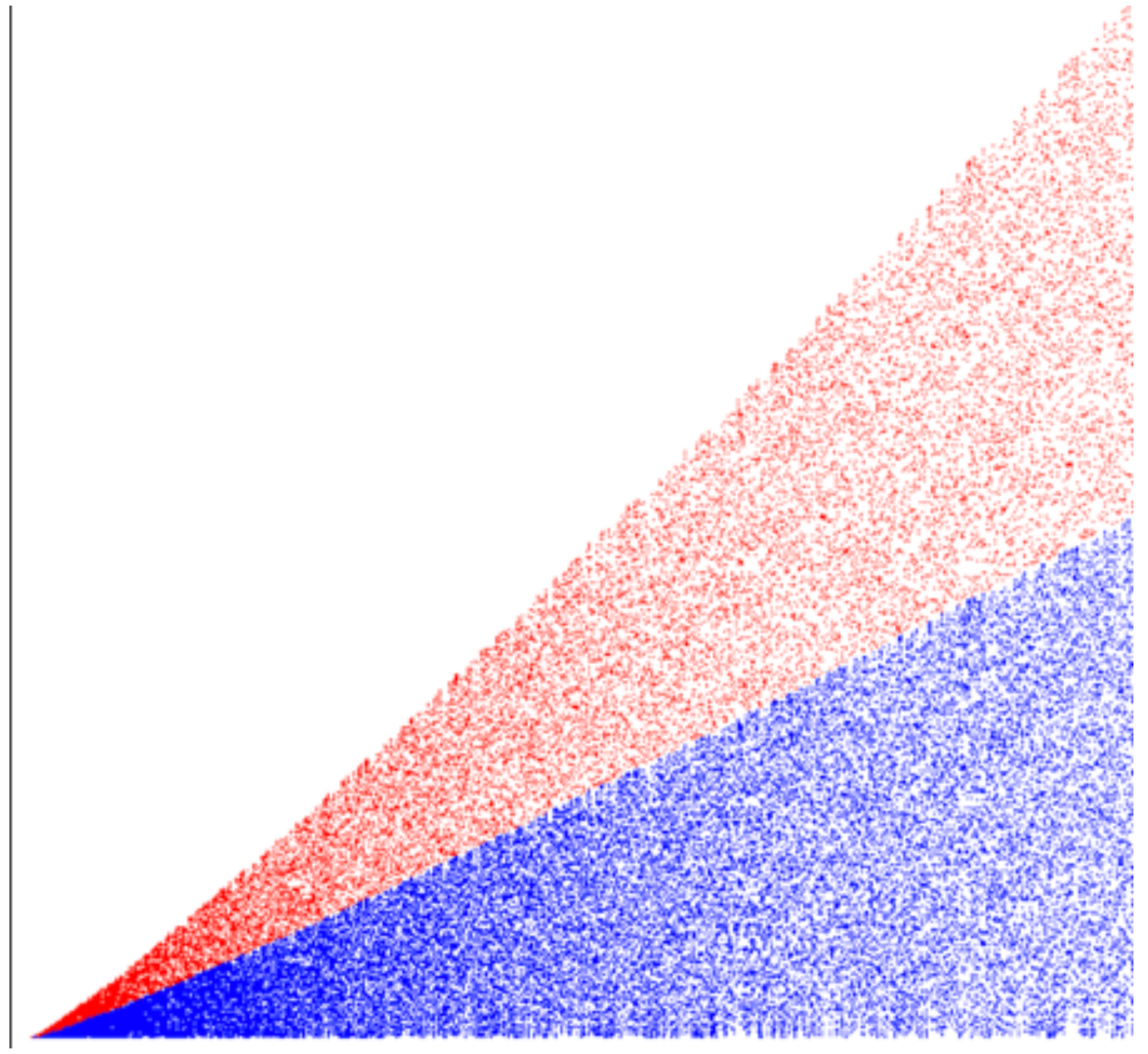
Studied by Russ Cox, Michael De Vlieger, Martin Ehrenstein, Hans Havermann, Rémy Sigrist, Allan C. Wechsler, and others

A350877 (Sisyphus)

a(1)=1; if even, divide by 2, if odd add next prime

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
a(n)	1	3	6	3	8	4	2	1	8	4	2	1	12	6	3	16	8	4	2	1	18	9	28	14	7	30	15
	2	3		5				7				11			13					17		19			23		





30K terms, slope of upper line around 7,
red = terms following an odd term

The big open question: does every number appear?

After 10^9 terms missing 36, 72, …

36 is part of a descending chain that ends with a(77534485879) = 9
and starts with a(77534485842) = 1236950581248
after adding the prime 677121348413.

a(17282073747557) = 97 ends a descending chain that starts with
a(17282073747516) = 21330525578544
after adding the prime 183236837077571. [Martin Ehrenstein]

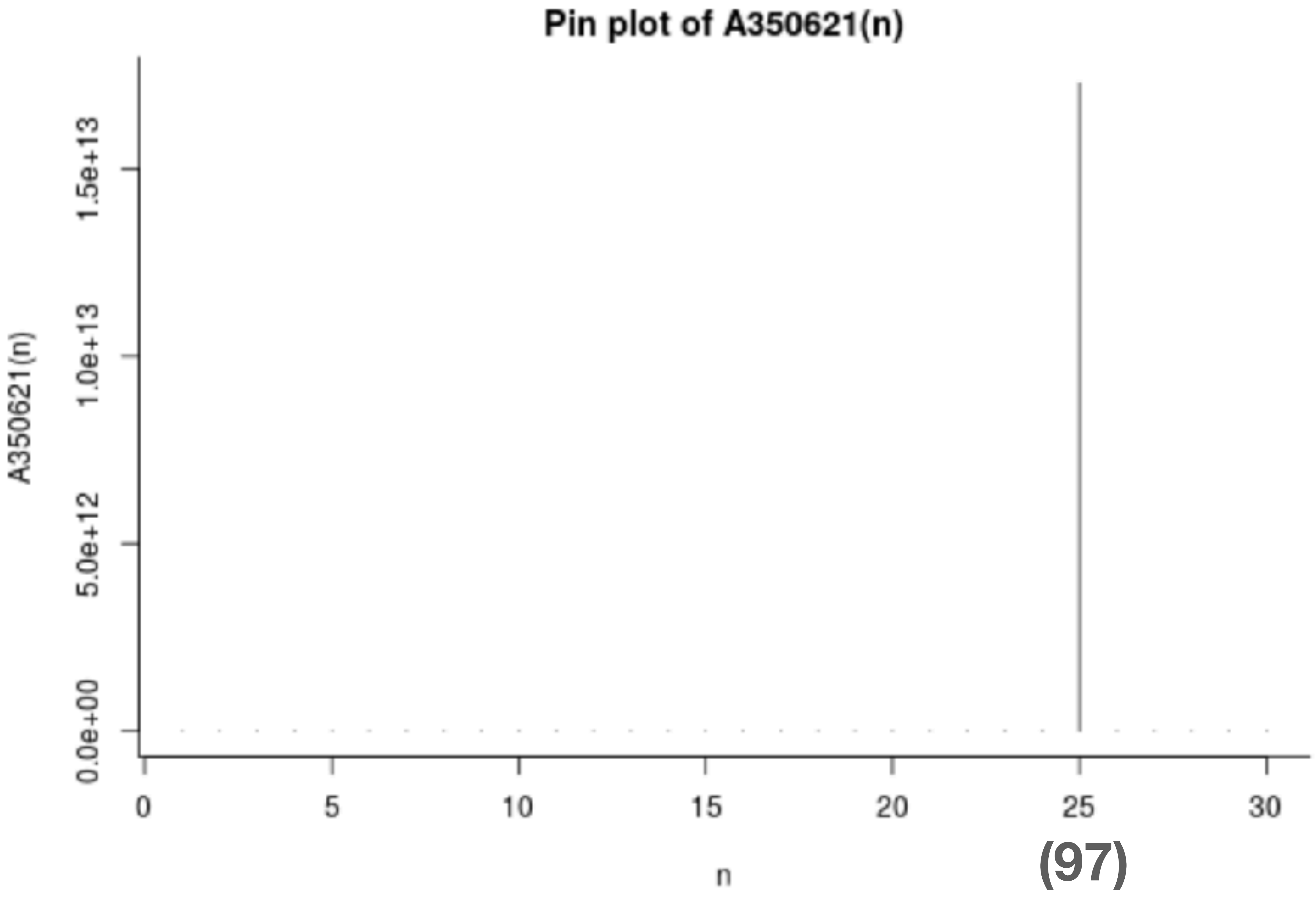
Conjecture: On naive probabilistic grounds, all integers should eventually appear.
An up-step is always immediately followed by a down-step, and then, on average,
by one more down-step. So we expect that every third step will be an up-step,
by the next prime number, which will be around p(n/3).
So the sequence will spend a lot of its time between p(n/3)/3 and 4p(n/3)/3.

[Allan C. Wechsler]

A350877 (Sisyphus), continued

When n-th prime appears for the first time, or -1 if it never appears.

7, 2, 71, 25, 30, 345, 161, 148, 51, 34, 48, 63, 234, 40, 126, 73, 135, 192, 454, 97, 78, 24841, 433, 85, 17282073747557, 322, 102, 106544217, 207, 556, …



A350621

Martin Ehrenstein

The Biggest Number of the Year?

(Not a serious question!)

Is it from the Sisyphus sequence?

`a(17282073747516) = 213305255788544`

Is it Augusto Santi's 2269429312765395470820 in A351871 ?

Is it Pavel Kropitz's new lower bound on Busy Beaver(6) ?

BB(6) > 10^^15 (A tower of 10's of height 15)

A060843: BB(1) = 1, BB(2) = 6, BB(3) = 21, BB(4) = 107, BB(5) = ?, BB(6) = ?

No! 10^^log n from Kaprekar's Junction Numbers (see below)

Kaprekar’s Junction Numbers

(with Max Alekseyev)

(To appear in Journal of Combinatorics and Number Theory,
December, 2022; arXiv:2112.14365, 2021)

The Mathematics of the New Self Numbers

(The first six monthly Report of the research work in mathematics done under U. G. C. scheme)

D. R. Kaprekar,

Price Rs. 1–50

1963

PREFACE

I have great pleasure in publishing this 1st six monthly report of my work. I am entirely devoted for the last 30 years in my original research on numbers and their peculiar properties. I could publish my works like " Demlo number " " cycles of recurring decimals Volume I " " cycles of recurring decimals Volume II " before about 15 years, I got for it some money (some portion of actual cost) through the University of Bombay. Further I published my books like " The new constant 6174. " The puzzles of the self numbers " through some grant of the University of Poona, I have expressed my indebtedness to the two Universities in the preface of those books. Further, also published about 20–30 articles on my researches in several Indian and foreign mathematical Journals. Now I am very glad to get completely devoted for my work as the University grands commission has given me a fine grant for continuing my work.

I am much grateful to Dr. D. S. Kothari, Dr. Ram Behari, Professor Hansaray Gupta and Several others who were kind enough to sanction this grant for me and I am sure I will publish various new interesting results in future.

This is the first six monthly report of my work. Though it was submitted in about February 1963. It could not be published earlier for various difficulties. The other six monthly reports will also follow soon.

Again expressing my gratefulness to all the authorities of the University grands commission at Delhi and with best regards to all of them. I declare this printing copy of the 1st report open to all on 1-11-63.

I remain
your truthfuly
D. R. Kaprekar.

All remarks over this work may please be sent to the address.

D. R. Kaprekar,
311 Devlali Camp.
DEVLALI
1-11-63.

See A3052 for full scan

Near Nashik

Dattaraya Ramchandra Kaprekar (1905-1986)

s(v) = sum of digits of v

f(v) = v + s(v)

The number of generators of u: F(u) = number of v such that f(v) = u

f(100) = 100 + 1 = 101

f(91) = 91 + 10 = 101

F(101) = 2

Kaprekar's Self numbers: F(u) = 0: 1, 3, 5, 7, 9, 20, 31, 42, … (A3052)

Junction numbers: F(u) >= 2: 101, 103, 105, 107, 109, 111, 113, … (A230094)

K(n) = smallest number u such that F(u) = n.

K(1) = 0, K(2) = 101,

K(3) = 10^13 + 1 (Kaprekar, early 1960's)

Kaprekar (cont.)

K(n) = smallest number u such that F(u) = n.

K(1) = 0, K(2) = 101,

K(3) = 10^13 + 1 (Kaprekar, early 1960's)

v = 999 999 999 9901, s(v) = 11*9 + 1 = 100, v + s(v) = 10^13 + 1

v = 999 999 999 9892, s(v) = 11*9 + 10 = 109, v + s(v) = 10^13 + 1

v = 10^13, s(v) = 1, v + s(v) = 10^13 + 1.

K(4) conjectured to be 10^24 + 102 by Kaprekar and Gunjikar,
In 1963, Narasinga Rao conjectured
K(5) = 10^1111111111124 + 102,
and gave upper bounds for K(6), K(7), K(8), and K(16).

Remarkably, all these conjectures and bounds are in fact the exact values of K(n).

Kaprekar (cont.)

We will show that K(1) - K(9) are

A6064

$$0,\ 101,\ 10^{13}+1,\ 10^{24}+102,\ 10^{1111111111124}+102,\ 10^{2222222222224}+10^{13}+2$$

$$10^{(10^{24}+10^{13}+115)/9}+10^{13}+2,\ 10^{(2\cdot 10^{24}+214)/9}+10^{24}+103,$$

$$10^{(10^{1111111111124}+10^{24}+214)/9}+10^{24}+103,\ \ldots$$

K(30) is

```
10^((2*10^((10^((2*10^((2*10^2+16)/9)+2*10^2+14)/9)+10^((10^((2*10^2+16)/
9)+10^((10^2+17)/9)+10^2+15)/9)+10^((2*10^2+16)/9)+10^((10^2+17)/9)+10^2+13)/
9)+2*10^((10^((2*10^2+16)/9)+10^((10^2+17)/9)+10^2+15)/9)+2*10^((10^2+17)/
9)+12)/9)+10^((10^((2*10^((2*10^2+16)/9)+2*10^2+14)/9)+10^((10^((2*10^2+16)/
9)+10^((10^2+17)/9)+10^2+15)/9)+10^((2*10^2+16)/9)+10^((10^2+17)/9)+10^2+13)/
9)+10^((10^((2*10^2+16)/9)+10^((10^2+17)/9)+10^2+15)/9)+10^((10^2+17)/9)+4
```

and K(100) is:

A6064(100) = K(100) =

10^((9*10^((2*10^((10^((10^((10^((2*10^2+16)/9)+10^((10^2+17)/9)+10^2+15)/9)+10^((2*10^((10^2+17)/9)+16)/
9)+2*10^((10^2+17)/9)+14)/9)+10^((2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+14)/
9)+2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+12)/9)+2*10^((2*10^((2*10^((10^2+17)/9)+16)/
9)+2*10^((10^2+17)/9)+14)/9)+2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+10)/
9)+2*10^((10^((10^((10^((2*10^2+16)/9)+10^((10^2+17)/9)+10^2+15)/9)+10^((2*10^((10^2+17)/9)+16)/
9)+2*10^((10^2+17)/9)+14)/9)+10^((2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+14)/
9)+2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+12)/9)+2*10^((2*10^((2*10^((10^2+17)/9)+16)/
9)+2*10^((10^2+17)/9)+14)/9)+2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+19)/
9)+10^((2*10^((10^((10^((10^((2*10^2+16)/9)+10^((10^2+17)/9)+10^2+15)/9)+10^((2*10^((10^2+17)/9)+16)/
9)+2*10^((10^2+17)/9)+14)/9)+10^((2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+14)/
9)+2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+12)/9)+2*10^((2*10^((2*10^((10^2+17)/9)+16)/
9)+2*10^((10^2+17)/9)+14)/9)+2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+10)/
9)+10^((10^((10^((10^((2*10^2+16)/9)+10^((10^2+17)/9)+10^2+15)/9)+10^((2*10^((10^2+17)/9)+16)/
9)+2*10^((10^2+17)/9)+14)/9)+10^((2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+14)/
9)+2*10^((2*10^((10^2+17)/9)+16)/9)+2*10^((10^2+17)/9)+12)/9)+10^((2*10^((2*10^((10^2+17)/9)+16)/
9)+2*10^((10^2+17)/9)+14)/9)+10^((2*10^((10^2+17)/9)+16)/9)+10^((10^2+17)/9)+6

about 10^^9

We give a set of recurrences which generate the sequence K(n) for any base b.
These recurrences depend upon an apparently new recurrence for F(u).

Two canonical examples:

Base 10, K(4) = 10^24 + 102
The 4 generators are
10^24 + 91, 10^24 + 100; 10^24 - 107, 10^24 - 98.

Base 2, K(4) = 4102
The 4 generators are
2^12+3, 2^12+4; 2^12-4, 2^12-5.

To check, use
s(2^m - u) = m - s(u-1) ,
and in base b, s(b^m - u) = (b-1)m-s(u-1).

2^7 - 5
= (2^7-1) - (5-1)
= 1111111 - 100
= 1111011
so
s(2^7 - 5) = 7 - wt(5-1)
= 7 - 1 = 6
and
2^7 - 5 + 6
= 2^7 + 1

The Key Recurrence in the Binary Case

Lemma: Every $u > 2$ has a unique representation as

$u = 2^m + 1 + k$ $(0 \le k \le 2^m - 1)$ .... (*)

Theorem: If u is given by (*), then

$Gen(u) = \{ 2^m + v : v \text{ in } Gen(k)\}$

$\cup \{ 2^m - 1 - v : v \text{ in } Gen(m - k - 2)$.

Check: $f(2^m+v) = 2^m+v+1+s(v) = 2^m+1+k$, right!

Hence:

$F(u) = F(k) + F(m-k-2)$.

In base b: $u = c(b^m+1) + k$,

$F(u) = F(k) + F((b-1)m-k-2)$.

Base 10: $u = 10^{13} + 1$, $k = 0$, $F(u) = F(0) + F(9.13-0-2) = F(0) + F(115) = \ldots = 1 + 1 + 1 = 3$.

For small bases the recurrences for K(n) are straightforward.

For base 2, K(1)=0, K(2)=5, K(3)=129, K(4)=4102, K(5)=2^136 + 6. Then:

$$E(n) = K\left(\left\lceil \frac{n}{2} \right\rceil\right) + K\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 2$$

A230302

$$K(n) = 2^{E(n)} + 1 + K\left(\left\lfloor \frac{n}{2} \right\rfloor\right)$$

A230303

But for bases b >= 4, to find K(n) we also need to know the values of

K_i(n) = smallest number v = i (mod b-1) such that F(v) = n,
and then K(n) = min_i K_i(n).

The recurrences get much more complicated.

Remark 1: For base b=5, the sequence of the best i's turns out to be essentially the classical Thue-Morse sequence A010060.
So we define generalized Thue-Morse sequences corresponding to other bases. See the paper for details.

Remark 2: K(n) = 10^^((log_2 n)+2)

The base-10 Thue-Morse sequence A239896

0, 2, 2, 4, 4, 4, 4, 6, 6, 6, 6, 6, 6, 6, 6, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 8, 3, . . .

LES: Lexicograpgically Earliest Sequences
(of distinct positive numbers such that…)

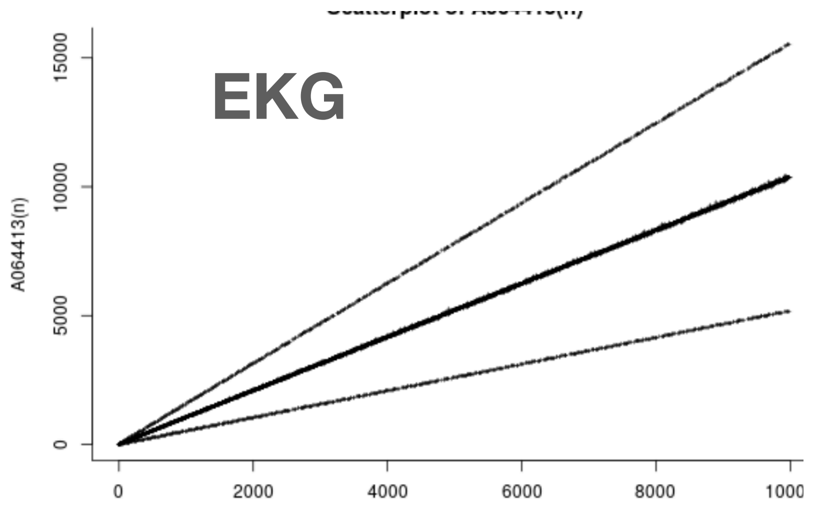
Lexicographic Order on Sequences of Nonnegative Integers

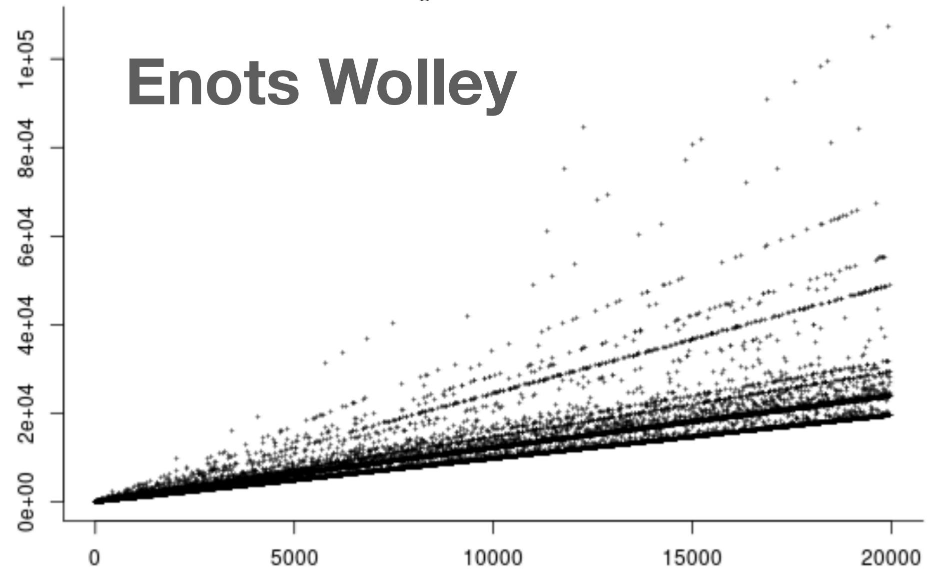
`(blank) <  0  <  1  <  2  <  3  <  4  <  ...`

1, 2, 4, 6, ... comes before 1, 2, 5, 6, ...

1, 2 (blank) comes before 1, 2, 0, 0, 5, ...

"LES" class of sequences: Lexicographically Earliest Infinite Sequence of Distinct Positive Numbers Such That ...





EKG sequence is a classical example: LES such that gcd(a(n-1), a(n)) > 1 for n>2

(A064413)

Enots Wolley: LES such that gcd(a(n-1), a(n)) > 1 and gcd(a(n-2),a(n)) = 1 for n>2

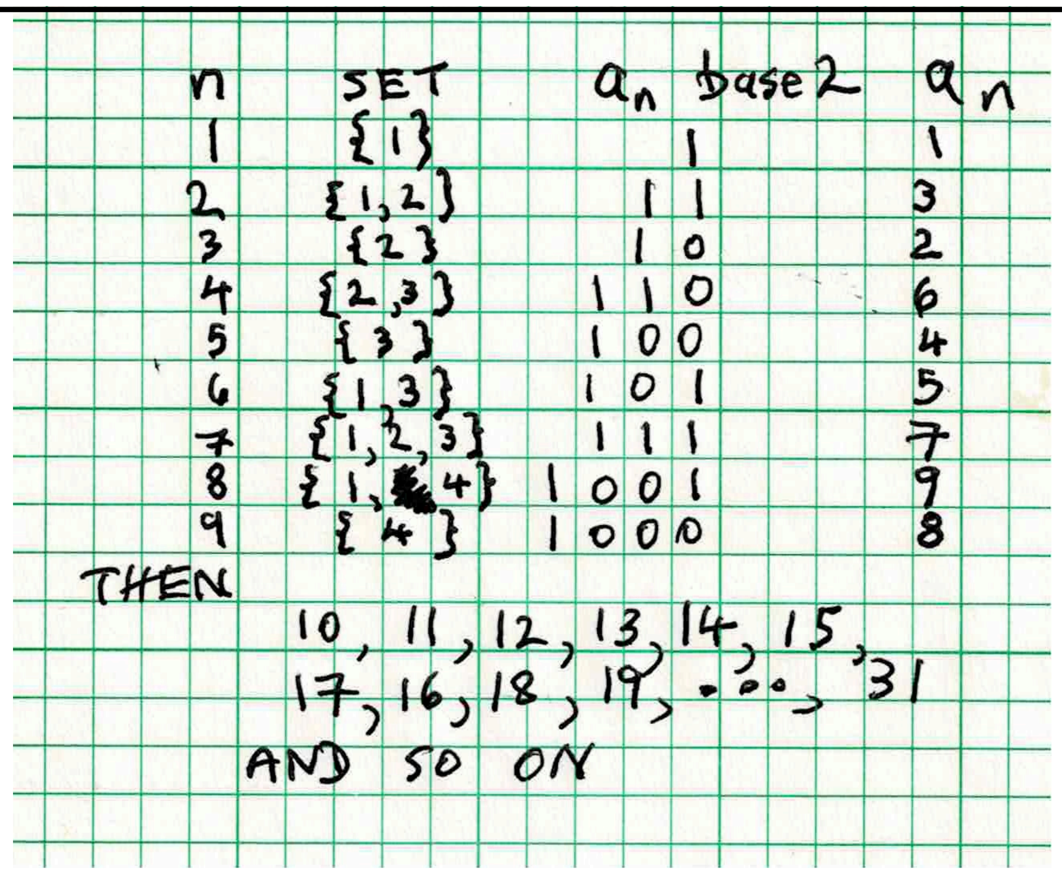
(A336957)

Set theory analogs of EKG etc.

Replace gcd(x, y) = 1 with $x \cap y \; = \; \varnothing$

Replace gcd(x, y) > 1 with $x \cap y \; \neq \; \varnothing$

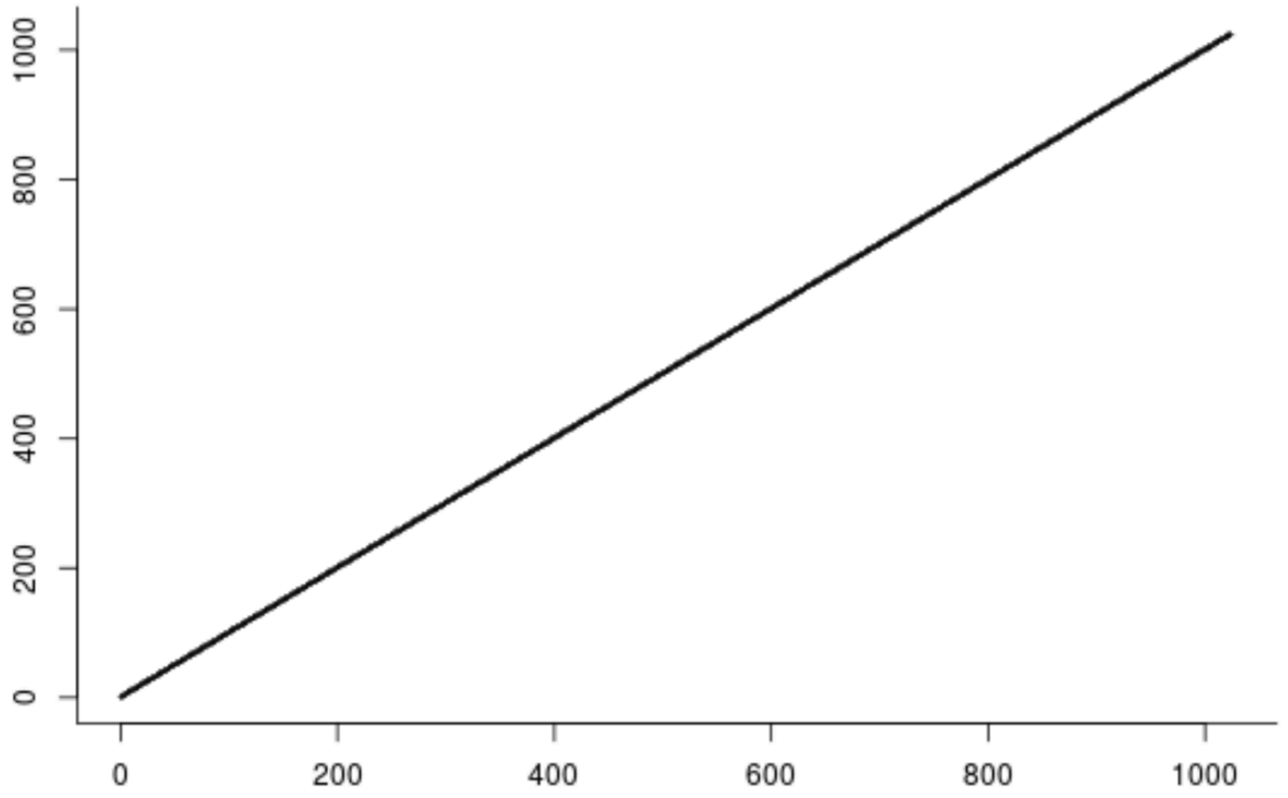
Set theory analog of EKG: $a_{n-1} \cap a_n \neq \varnothing$ as subsets of N



n	SET	a_n base 2	a_n
1	{1}	1	1
2	{1,2}	11	3
3	{2}	10	2
4	{2,3}	110	6
5	{3}	100	4
6	{1,3}	101	5
7	{1,2,3}	111	7
8	{1,4}	1001	9
9	{4}	1000	8

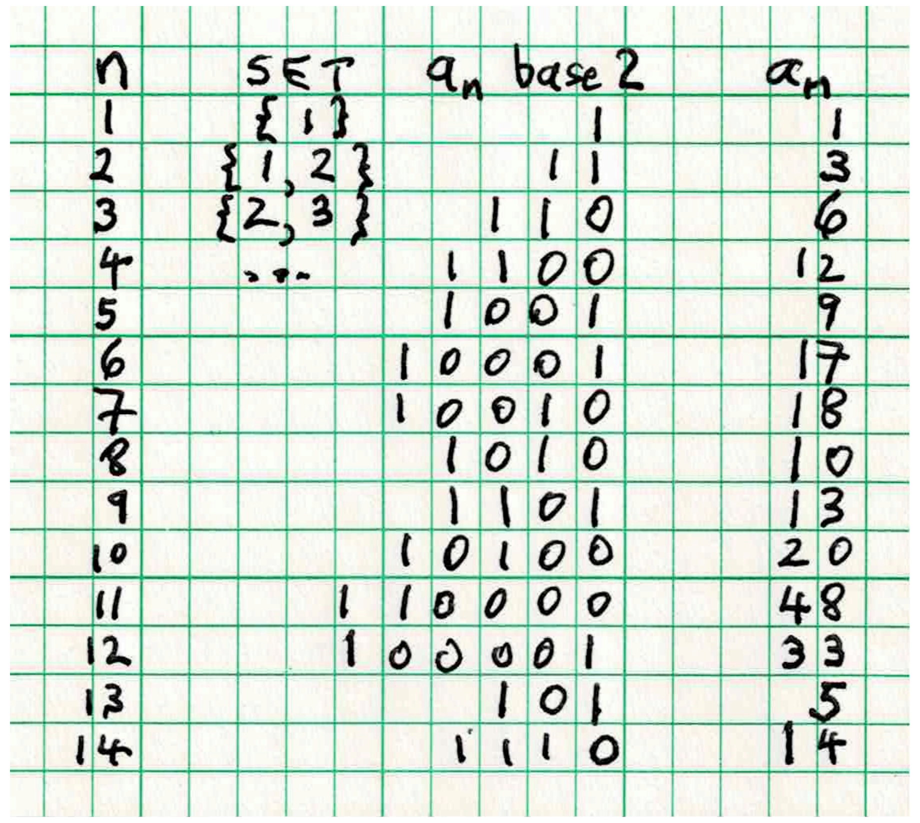
THEN 10, 11, 12, 13, 14, 15, 17, 16, 18, 19, ..., 31 AND SO ON

A115510



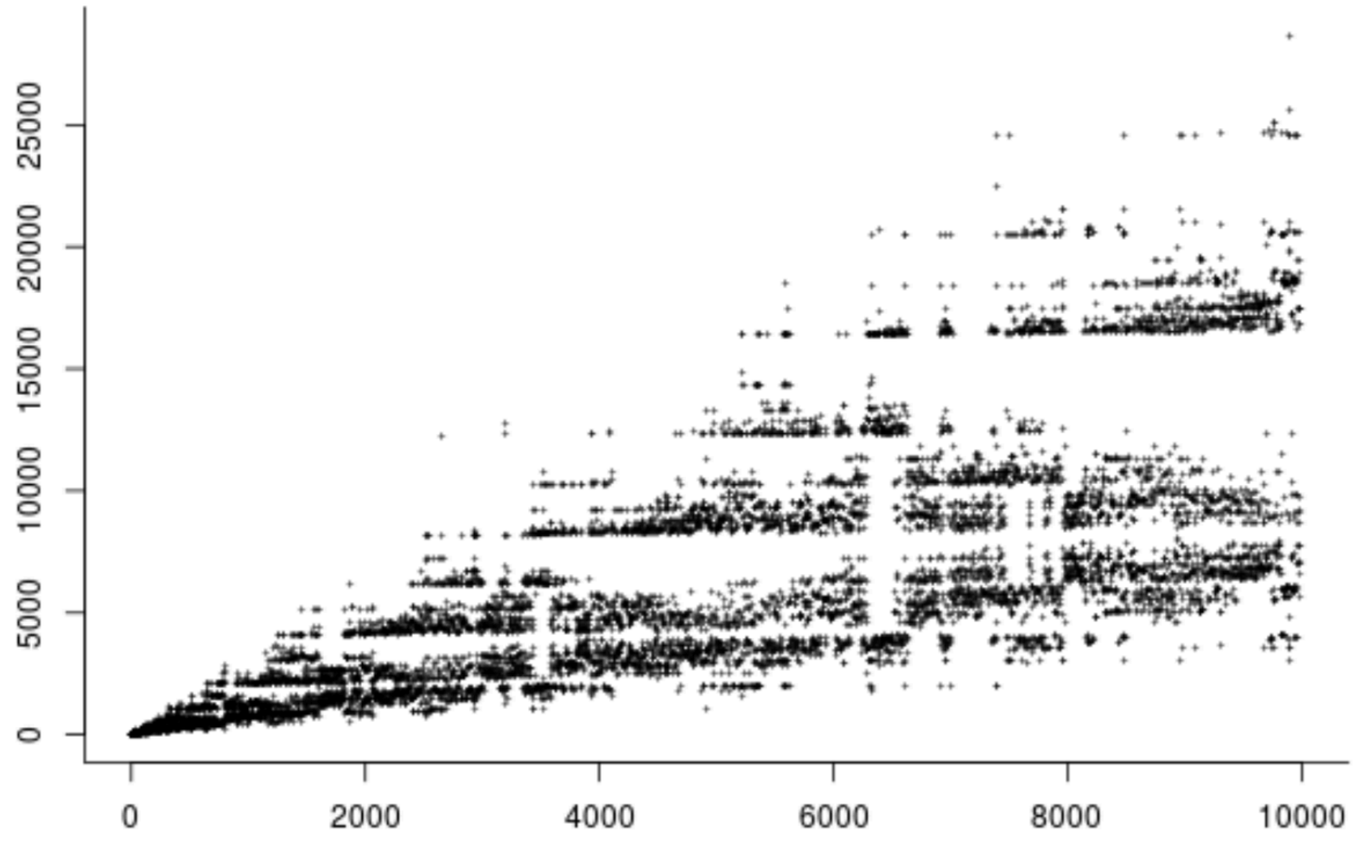
Easy Theorem: Every number appears

Set theory analog of Enots Wolley: add $a_{n-2} \cap a_n \; = \; \varnothing$ and $a_n \backslash a_{n-1} \; \neq \; \varnothing$



n	SET	a_n base 2	a_n
1	{1}	1	1
2	{1,2}	11	3
3	{2,3}	110	6
4	...	1100	12
5		1001	9
6		10001	17
7		10010	18
8		1010	10
9		1101	13
10		10100	20
11		110000	48
12		100001	33
13		101	5
14		1110	14

A338833



What ???

Theorem (Nathan Nichols): This is a permutation of { N excluding 2,4,8,16,... }

"Homework" 1. Check the proof. [I really hope someone - or some collaboration - will do this. I have not checked it myself. It needs to be done.]
2. Can the proof be adapted to show that Enots Wolley is a permutation of N ?

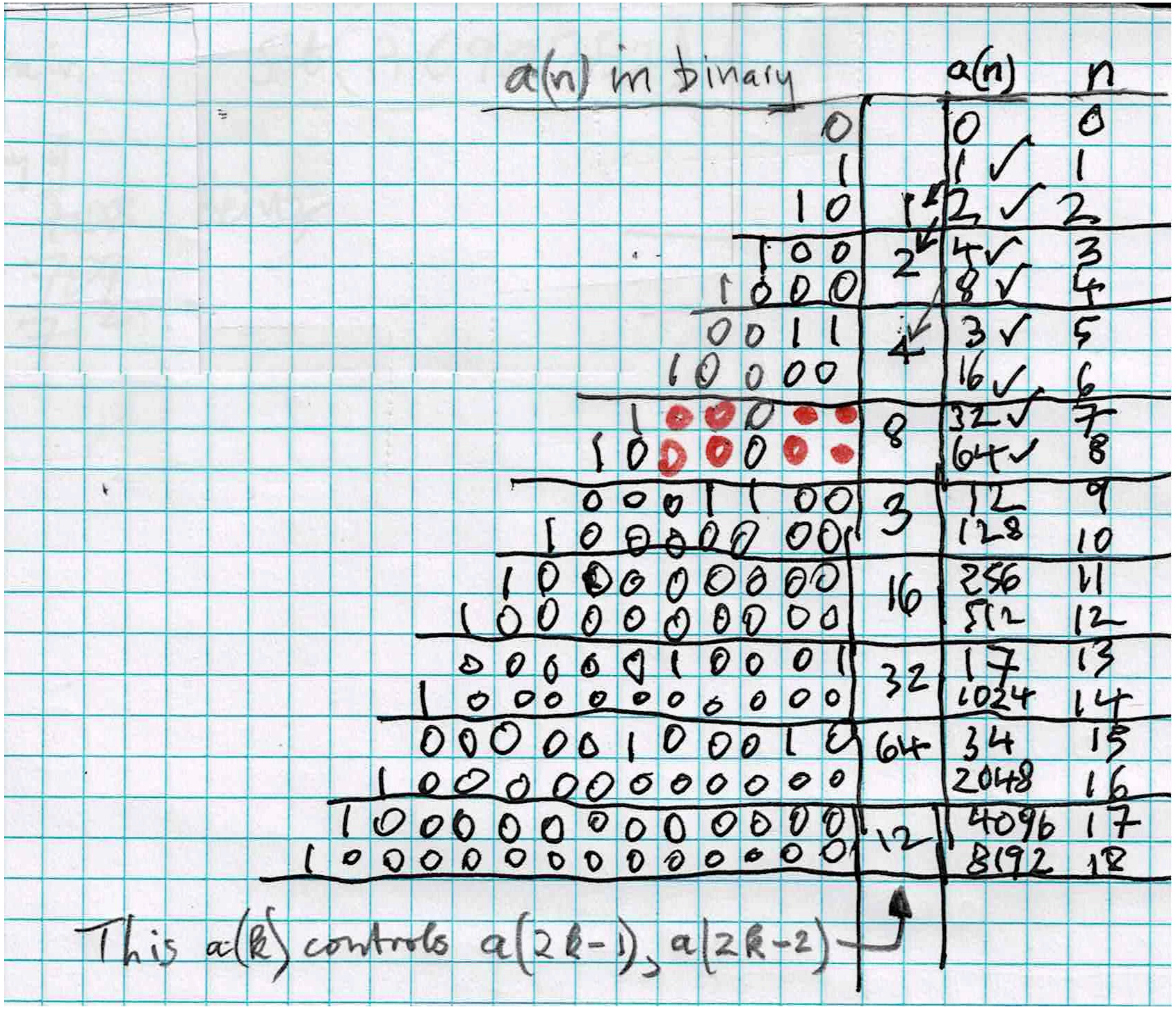
The Binary Two-Up Sequence

Based on joint work with Michael De Vlieger, Thomas Scheuerle, Rémy Sigrist, and Walter Trump

The binary two-up sequence (A354169) is the Lexicographically Earliest Sequence of distinct nonnegative numbers such that a(n) is "perpendicular" to the following n terms.

"Perpendicular" means
"binary expansions have disjoint sets of 1's"
$9 = 1001 \perp 6 = 110$

"Lexicographically Earliest":
1, 2, 4, … comes before 1, 2, 5, …



A354169 (cont.)

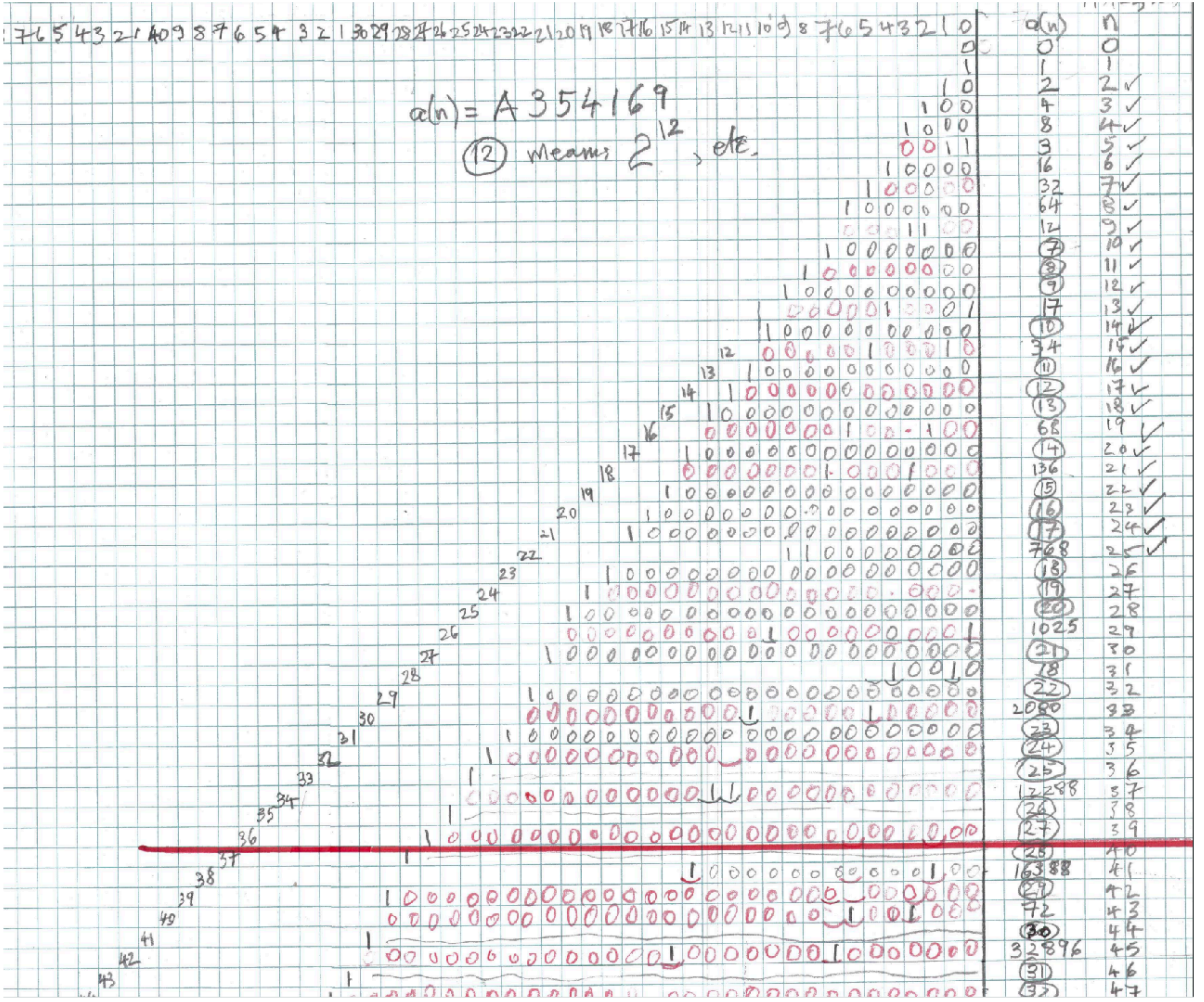
The algorithm for constructing the sequence

When you know a(k), append two terms:
a(2k-1) = minimum m not in sequence such that {m, a(k), a(k+1),…,a(2k-2} are perpendicular,
a(2k) = minimum m’ not in sequence such that {m’, a(k), a(k+1),…,a(2k-1} are perpendicular,

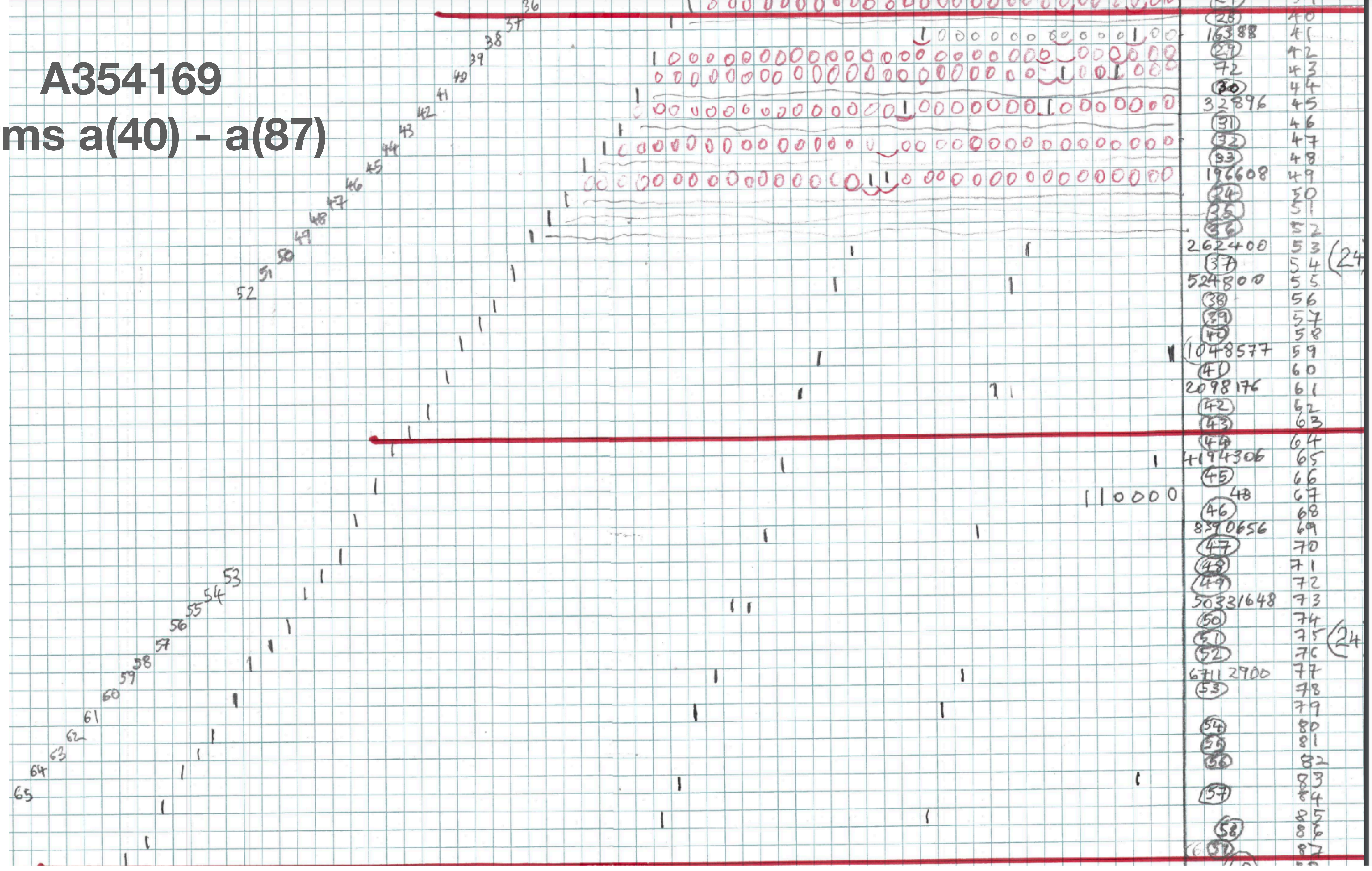
Given input a(0),…, a(k), this produces output a(0),…,a(2k).

Iterating, the output converges to A = A354169.

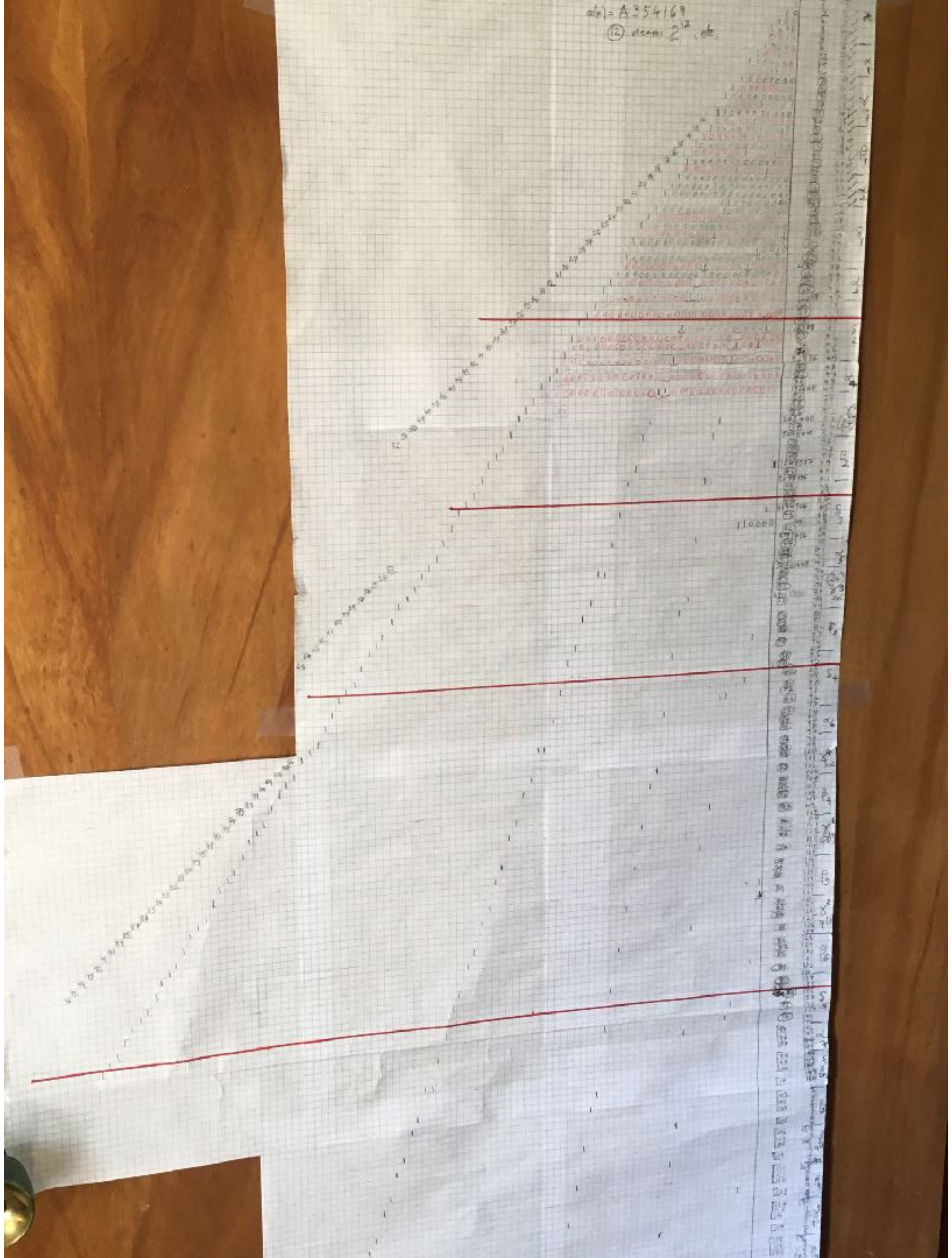
A354169
Terms a(0) - a(47)



A354169
Terms a(40) - a(87)



The “Dead Sea Scroll”
192 terms of A354169



24

24

48

48

A354169 (cont.) The Log File and Atoms

Log entries are: Aw means added a term of Hamming weight w
Fw means freed a term of Hamming weight w
[1] means 2^1, [3,6] means 2^3 + 2^6, etc.

The 5 types of atoms

Table 1

T	**=**	**A1A1F1A1A1F1 ,**	
U	=	A2A1F1A1A1F1 ,	
V	=	A2A1F2A1A1F1A2A1F1 ,	
W	=	A2A1F1A1A1F2A2A1F1 ,	
X	=	A2A1F2A1A1F1A2A1F2A2A1F1	.

An atom ends when there are exactly two free powers of 2

Table 2: Log file for $A354169$.

$A1$	found	$a(1) = [0]$	start of atom T
$A1$	found	$a(2) = [1]$	
$F1$	freed	$a(1) = [0]$	
$A1$	found	$a(3) = [2]$	
$A1$	found	$a(4) = [3]$	
$F1$	freed	$a(2) = [1]$	end of atom T
$A2$	found	$a(5) = [0,1]$	start of atom $U(0)$
$A1$	found	$a(6) = [4]$	
$F1$	freed	$a(3) = [2]$	
$A1$	found	$a(7) = [5]$	
$A1$	found	$a(8) = [6]$	
$F1$	freed	$a(4) = [3]$	end of atom $U(0)$
$A2$	found	$a(9) = [2,3]$	start of atom $V(-1)$
$A1$	found	$a(10) = [7]$	
$F2$	freed	$a(5) = [0,1]$	
$A1$	found	$a(11) = [8]$	
$A1$	found	$a(12) = [9]$	
$F1$	freed	$a(6) = [4]$	
$A2$	found	$a(13) = [0,4]$	
$A1$	found	$a(14) = [10]$	
$F1$	freed	$a(7) = [5]$	end of atom $V(-1)$
$A2$	found	$a(15) = [1,5]$	start of atom $W(-1)$
$A1$	found	$a(16) = [11]$	
$F1$	freed	$a(8) = [6]$	

T atom
U atom
V atom
W atom

A354169 (cont.)

The Main Theorem

Let S = T U V W U, and for k >= 1 let

$$R(k) \;=\; (VW)^{2^{k-1}-1}\, XU\, (VW)^{2^{k-1}-1}\, XU$$

Then A is the limit as k goes to infinity of S R(1) R(2) R(3) … R(k).

Also, for k >= 2, every pair of atoms XU in R(k) has the "ancestor property" that …

The proof uses a map Φ which is a local version of the map that defines A, and sends a string of atoms in A to a string of atoms in A with twice the number of terms.

When there is a term a(k) in the input the output contains two terms a(2k-1), a(2k), each being either the smallest subset of the free powers of 2 that has not yet appeared, or the next free power of 2.

Since the map Φ does not know what came before the start of the input string, we use two arguments to certify that the term we want to add has not appeared before.

(1) When 2^e is freed for the first time, no term 2^e + 2^f (f<e) has appeared;

(2) If the input contains a pair of atoms XU, the ancestor property saves us.

A354169 (cont.)

Proof of Main Theorem, (cont.)

We check that, under the map Φ,

S R(1)	goes to	S R(1) R(2),
V	goes to	VW
W	goes to	VW
X	goes to	VWXU
	hence	
R(k)	goes to	R(k+1), k >= 2

QED

Corollary: Every term of A354169 is the sum of at most two powers of 2.

Corollary: We can say exactly what the term are in each atom, and hence in A itself.
The formulas all involve A029744.

A354169 (cont.)

The number-theory versions

If “perpendicular” meant “relatively prime”,
we get A090252 and A354790.

Open Problem: Show their terms are
products of at most two primes.

If there was more time, I would also discuss:

Grant Olson’s LES A347113: still lacks a proof that every number appears.

Set-theory version of Enots Wolley LES, A338833: someone please check proof
that every number of weight >= 2 appears.

Rémy Sigrist’s Pascal Triangle with knight’s moves (A355320, A096608).

Colin Mallows’s conjectures about number of solutions to u+v+w+x = n, u^2+v^2+w^2+x^2 = n^2
(A278085).

Seiichi Manyama’s A351372: solutions to (x+y)^2 + (y+z)^2 + (z+x)^2 = 12 xyz.

The most wanted prime: 123…n. See A7908.

Most wanted formulas (see next slide)

Sequences with many terms, need formulas

2 **From the analysis of LES A090252:**

3, 1, 15, 33, 61, 97, 121, 129, 133, 245, 265, 389, 481, 485, 489, 2065, …

https://njas.blog/2022/06/03/the-two-up-sequence-a090252/

3 **Simple interior nodes in 1 X n grid, A334701:**

1, 6, 24, 54, 124, 214, 382, 598, 950, 1334, … **Have 500 terms**

Also A292104.

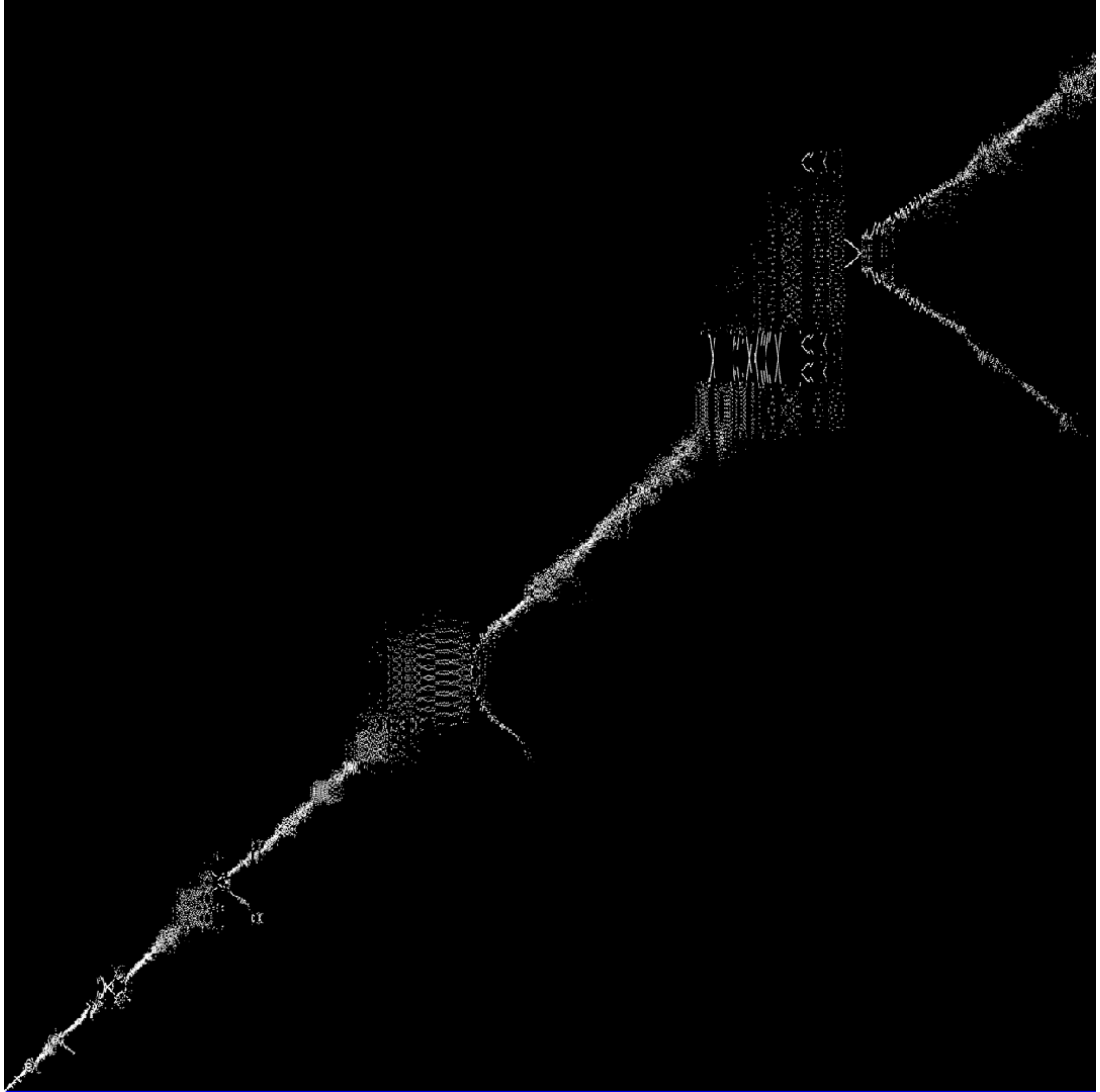
4 **Also A067151, A349784, A350606.**

5 **Look for words “It would be nice to have a formula …” in OEIS entries.**

The Scariest Sequence of 2022

A357082

20K terms



A357082

70K terms