The Remarkable Sequences of Éric Angelini

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In Memoriam Éric Angelini (Sep. 12 1951 - Sep. 27 2024)

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1700 sequences, brilliant, clever, surprising, witty, during 2004-2024.
One of my favorite contributors, and a friend for 20 years.
Éric said that when he discovered the OEIS he thought it was the eighth wonder of the world. He will be greatly missed

Contents: “Which Terms are Primes?”, The Jungfrau, Solar Flares, Oulipo, Even Digit Next Bigger, “Look Left”, Delete Repeated Digits, The Rigidity of the Okapi.

[Briefly: Choix de Bruxelles, Sisyphus, Comma Sequence]

“Which Terms are Primes?”

That’s the definition!   It hardly seems enough to specify a sequence, but it is.

The full definition is: The Lexicographically Earliest infinite Sequence
(“LES’)
of distinct positive numbers
that describes the positions of its prime terms:

2, 3, 5, 1, 7, 8, 11, 13, 10, 17, …
A121053, E.A., 2006

a(1) can’t be 1, because that would imply 1 is a prime. But a(1) = 2 seems to work, therefore a(1) IS 2. This implies a(2) is a prime, so take a(2) = 3, which implies a(3) is a prime, so take a(3) = 5 - the primes appear in order.
a(4) is now free, and its smallest possible value is 1 (and a(1) IS a prime).
Now 4 can’t appear, because that would say a(4) = 1 is a prime, which is false.
And so on.

After 18 years, the following is a new formula, with proof.

a(n) = A121053 The tableau:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
a(n)	2	3	5	(1)	7	(8)	11	13	(10)	17	19	(14)	23	29	(16)	31	37	(20)	41	43	(22)	47	53	(25)	59	(27)
Yes	1	2	3		5		7	8		10	11		13	14		16	17		19	20		22	23		25	
No				4		6			9			12			15			18			21			24		26
smn	1	1	1	6	6	9	9	9	12	12	12	15	15	15	18	18	18	21	21	21	24	24	24	26	26	28

n	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
a(n)	61	(30)	67	71	73	(33)	79	(35)	83	(38)	89	97	(40)	101	103	(44)	107	109	(46)	113	127	(49)	131	(51)	137	(54)
Yes	27		29	30	31		33		35		37	38		40	41		43	44		46	47		49		51	
No		28				32		34		36			39			42			45			48		50		52
smn	28	32	32	32	32	34	34	36	36	39	39	39	42	42	42	45	45	45	48	48	48	50	50	52	52	55

YES = indices of primes = A377898, must be in sequence NO = indices of composite terms, must not be in sequence

smn = smallest (legal) missing composite number

◡ = PRIMES ◯ = C_{2t} ◯ = C_{2t+1}

A121053

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
a(n)	2	3	5	1	7	8	11	13	10	17	19	14	23	29	16	31	37	20	41	43	22	47	53	25	59	27
Yes	1	2	3		5		7	8		10	11		13	14		16	17		19	20		22	23		25	
No				4		6			9			12			15			18			21			24		26
smn	1	1	1	6	6	9	9	9	12	12	12	15	15	15	18	18	18	21	21	21	24	24	24	26	26	28

n	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
a(n)	61	30	67	71	73	33	79	35	83	38	89	97	40	101	103	44	107	109	46	113	127	49	131	51	137	54
Yes	27		29	30	31		33		35		37	38		40	41		43	44		46	47		49		51	
No		28				32		34		36			39			42			45			48		50		52
smn	28	32	32	32	32	34	34	36	36	39	39	39	42	42	42	45	45	45	48	48	48	50	50	52	52	55

Theorem: Let p(n) = prime(n), c(n) = composite(n), $\pi(n)$ = PrimePi(n) and a(n) = A121053(n)

Then if n = p(i) or c(2t+1), a(n) = p(k), where k = floor((n+$\pi(n)$)/2), otherwise n = c(2t) and a(n) = c(2t+1).

Proof: (cf. tableau on previous slide)

If n = p, before a(n) there are pi(n)-1 primes, all primes from c(odd) < n, and 1, a total of pi(n) + floor((n-pi(n))/2) = floor((n+pi(n))/2) = k (say) earlier primes, and so a(n) = p(k).

If n = c(2t+1), same argument.

Otherwise n = c(2t), and a(n) is composite, smn = n = c(2t). So a(n) = next composite after c(2t), which is c(2t+1). QED

Corollary: The terms in A121053 consist of the primes and composites with odd subscripts, except change c(1) = 4 to 1.

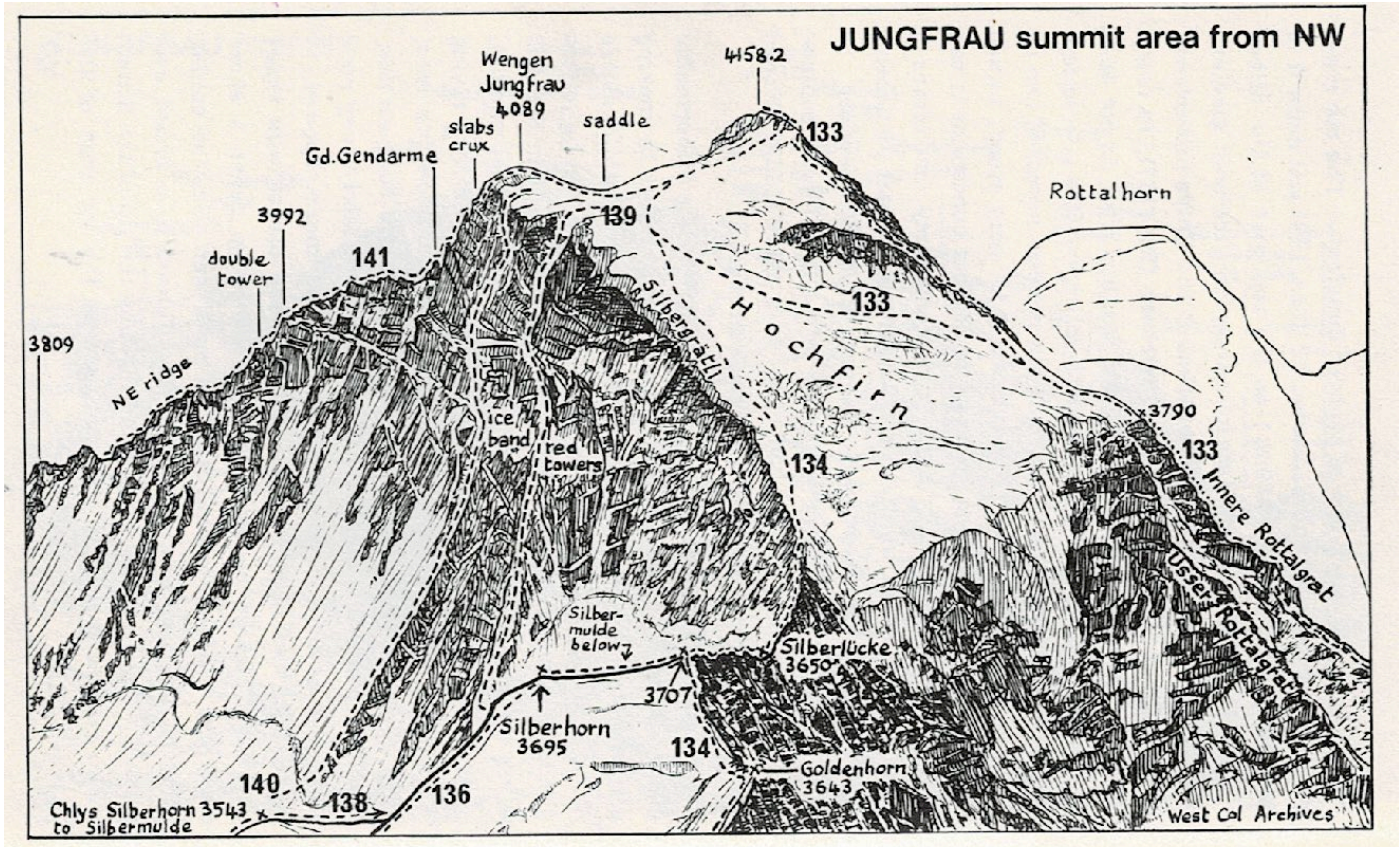
Corollary (Dean Hickerson, 2006): Density of primes in A121053 is 1/2.

If change “Primes” in definition of A121053 to “1 union primes” we get A377901, a 20th-century analog. If change “prime” to “odd” we get A079313, or to “even” we get A080032. For more examples, see table in A379051.

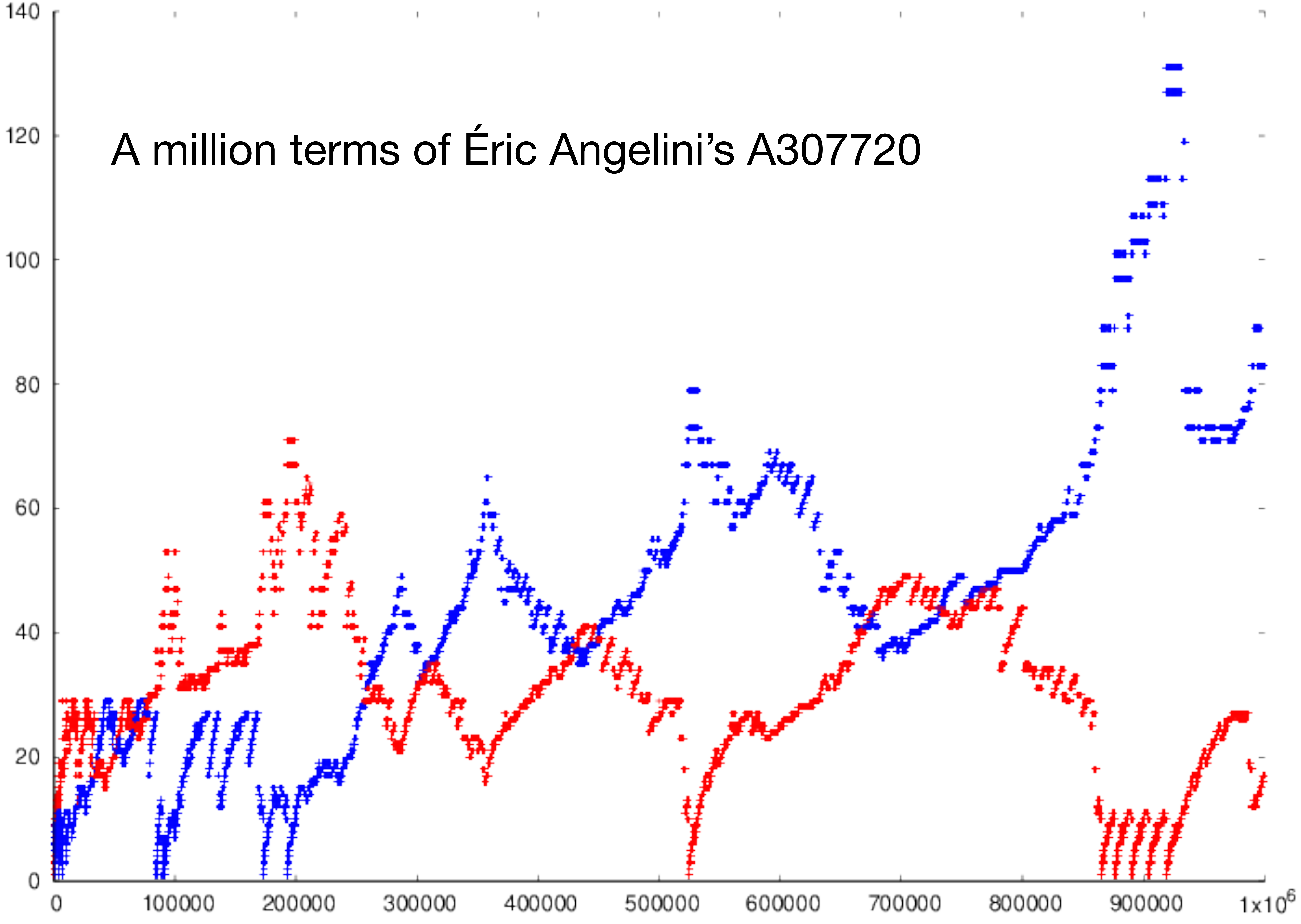
The Jungfrau

A307720

Éric Angelini and Jean-Marc Falcoz, April 24 2019



A million terms of Éric Angelini's A307720



[William Cheswick]

The Jungfrau Sequence (continued)

Definition: LES sequence A = {a(n): n >= 1} of positive numbers such that in the sequence B = {a(n).a(n+1): n >= 1} 1 appears once, 2 twice, 3 thrice, 4 four times, etc.

	n	1	2	3	4	5	6	7	8	9	10	11
A:	a(n)	1	1	2	1	3	1	3	2	2	2	2
B:	a(n)a(n+1)	1	2	2	3	3	3	6	4	4	4	4

A307720
A307730

A: 1, 1, 2, 1, 3, 1, 3, 2, 2, 2, 2, 2, 3, 2, 3, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 2, 4, 2, 4, 2,

B: 1, 2, 2, 3, 3, 3, 6, 4, 4, 4, 4, 6, 6, 6, 6, 6, 9, 9, 9, 9, 9, 9, 9, 9, 9, 12, 8, 8, 8, 8, 8, 8,

A remarkable sequence: Simple definition. 30+ derived sequences, all new.
An isolated component in the Great OEIS Graph.
Every number eventually appears, but slowly. After 100000 terms, 32 still missing(*).
2024 does not appear in A until term n = 855317952137.
Theorem: Greedy algorithm works. No backtracking needed.
Average order of a(n) not known. Average position of prime(n) not known.
Worth listening to: suggestive of early Techno, like Kraftwerk
Left and right hands switch at irregular intervals!

(*) 32 appears at n = 106968

When 5 finally appears it is immediately followed by 7. Under certain conditions, the primes appear in pairs, or even large clumps.

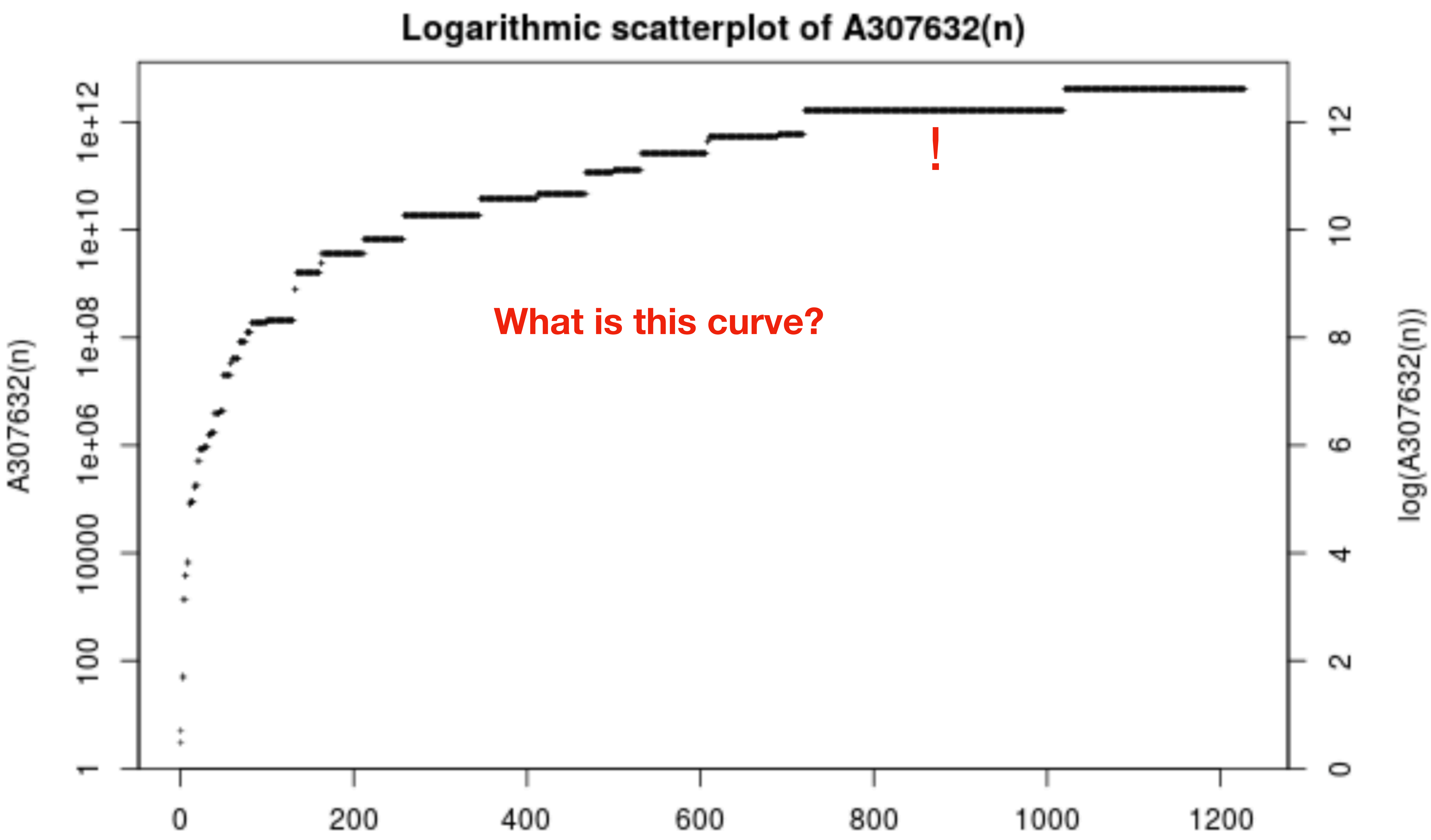
n	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
a(n)	4	3	5	1	5	1	5	1	7	1	7	1	7	1	7	2
b(n)	12	15	5	5	5	5	5	7	7	7	7	7	7	7	14	

If we color the point a(n even) red, and a(n+1, odd) blue, then the usual behavior (as seen here) is L hand (red) < R hand (blue). But every so often, the hands switch!

(see second slide)

Reminiscent of the Great Prime Race (Granville) betwen 4k+1 primes and 4k+3 primes. Probably same thing happens here; leads swap infinitely ofter with ever-increasing gaps. It would be nice to know more.

When prime p first appears in A (A307632)



Solar Flares

or “Digit Strean Unchanged by Digit Sums”

E.A. and Hans Havermann, April 2018. A302656

Let D(n) = digitsum(n). E.g. D(109) = 10.
Definition: S is LES infinite sequence of distinct positive numbers such that S and D(S) have same sequence of digits.

S = 1 2 3 4 5 6 7 8 9 x y …
D(S) = 1 2 3 4 5 6 7 8 9 D(x) D(y) …

Distinct implies x >= 10. But x = 10 implies D(y) begins with 0 , NO!
x = 11 fails because D(11) = 2, etc. x = 109 is smallest number that seems to work.

S = 1 2 3 4 5 6 7 8 9 109 y …
D(S) = 1 2 3 4 5 6 7 8 9 10 9 …

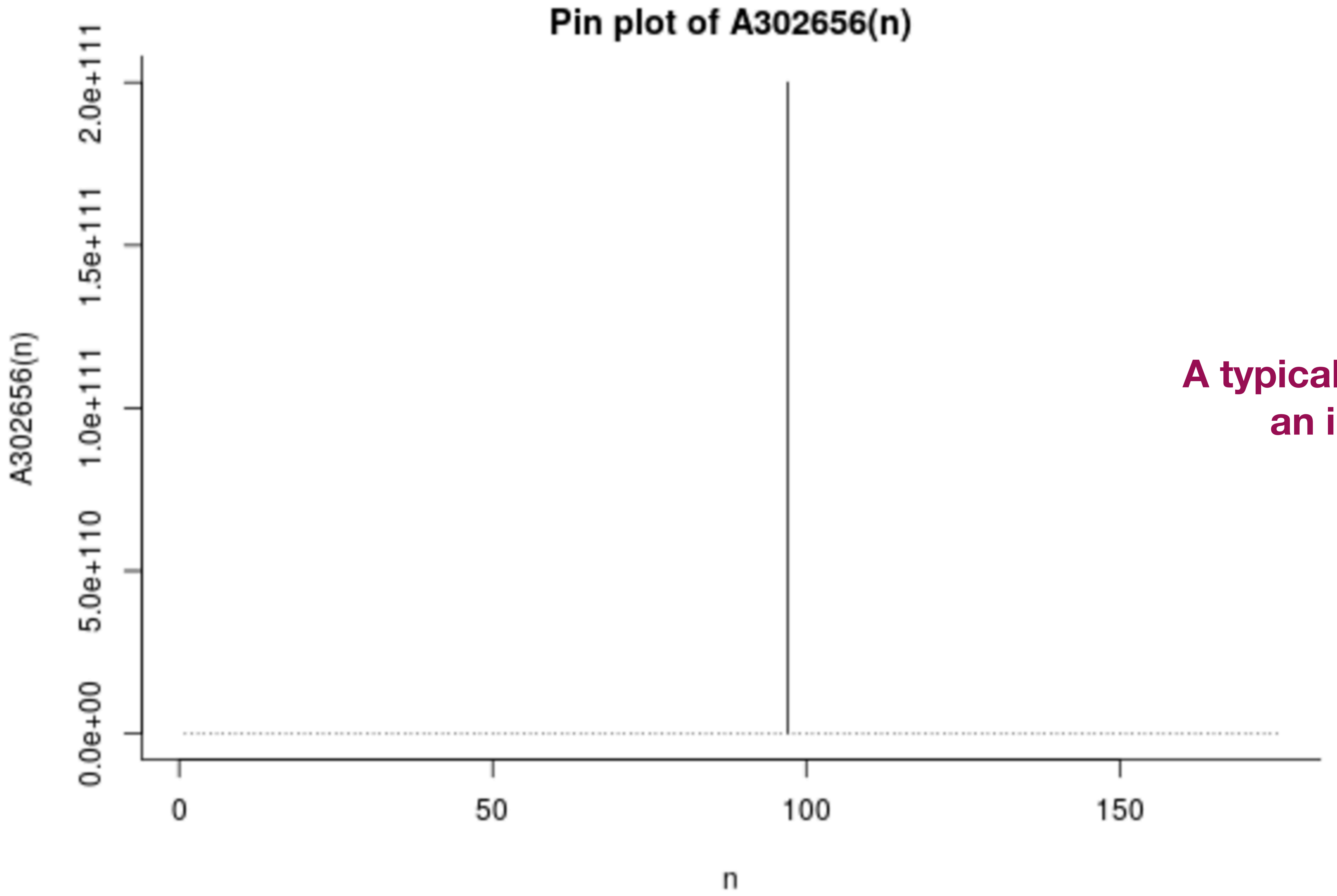
and y = 18 seems to work, and in fact does work.

Here M = 9

The algorithm:

Let M = unmatched portion of digit stream.
D(y) must match a prefix of M, perhaps all of M,
D(y) must not leave a leading 0 behind when deleted from M, must not violate the stream-of-digits constraint, and y must be new and minimal

Solar Flares (cont.) A302656: The first 180 terms!



A typical Angelini graph,
an inverted T !

How the huge numbers arise

	a(n) A302656,			D(a(n)) A376769,		DIGIT STREAM A376771					
n	1	2	3	4	5	6	7	8	9	10	11
a(n)	1	2	3	4	5	6	7	8	9	109	18
D(a(n))	1	2	3	4	5	6	7	8	9	10	9
n	12	13	14	15	16	17	18	19		20	
a(n)	10	17	19	89	100	27	26	36	199	999	999 999
D(a(n))	1	8	10	17	1	9	8	9		100	
n	21	22	23	24	25	26	27	28	29	30	31
a(n)	11	16	20	15	12	24	199	45	54	63	72
D(a(n))	2	7	2	6	3	6	19	9	9	9	9
n	50	51	52	53	54	55	56		57		
a(n)	31	33	21	25	110	35	1000	999	999999		
D(a(n))	4	6	3	7	2	8	1		90		
n	90	91	92	93	94	95	96		97		98
a(n)	120	399	799	10000	46	201	41	1	99…9		234
D(a(n))	3	21	25	1	10	3	5		1000	111 9's	

A302656
A376769

$2 \cdot 10^{11} - 1$

$2 \cdot 10^{111} - 1$

Where we see $2 \cdot 10^{(10^k-1)/9} - 1$

A377904

k	n	$a(n) = 2 \cdot 10^{\frac{10^k-1}{9}} - 1$
0	1	$1 = 2 \cdot 10^0 - 1$
1	14	$19 = 2 \cdot 10^1 - 1$
2	20	$199999999999 = 2 \cdot 10^{11} - 1$
3	97	$19^{(111)} = 2 \cdot 10^{111} - 1$
4	176	$19^{(1111)} = 2 \cdot 10^{1111} - 1$
5	396	$2 \cdot 10^{11111} - 1$
6	463	$2 \cdot 10^{111111} - 1$
7	1918	$2 \cdot 10^{1111111} - 1$
8	1984	$2 \cdot 10^{11111111} - 1$
9	2278	$2 \cdot 10^{(10^9-1)/9}$

These are the record high points in A302656 for k >= 2

What is this sequence A377904 ?

Thanks to Michael S. Branicky (1982 terms) and Dominic McCarty (10000 terms) for extending A302656 !

The Solar Flares Sequence (continued)

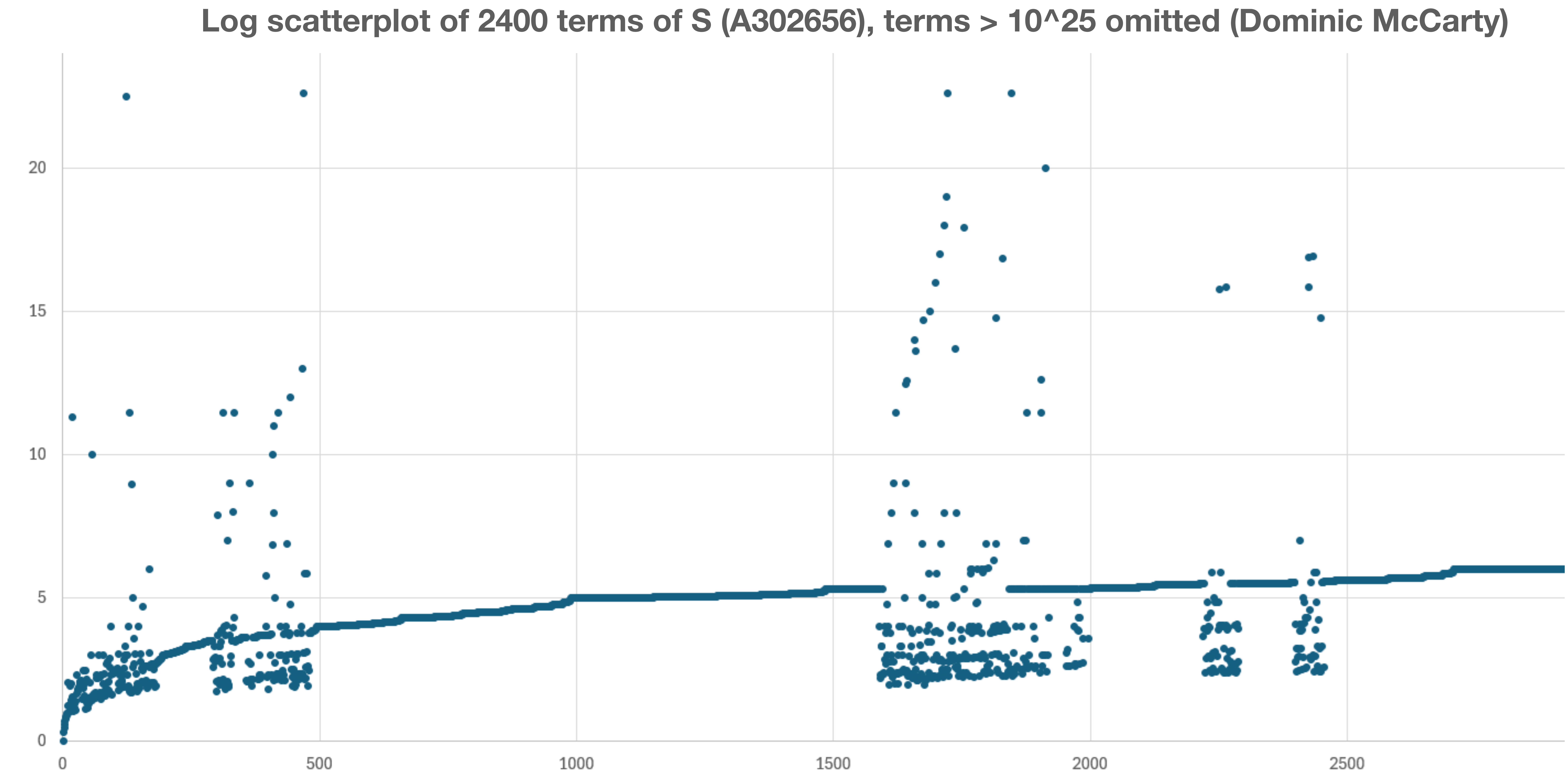
Where the huge spikes appear:

k = 3, the spike is a(97) = 1999…999 (with 111 nines, 111 = (10^k-1)/9),

caused by 1000 = 10^k in S at n = 56,

and by 1000 41 steps later in D(S)

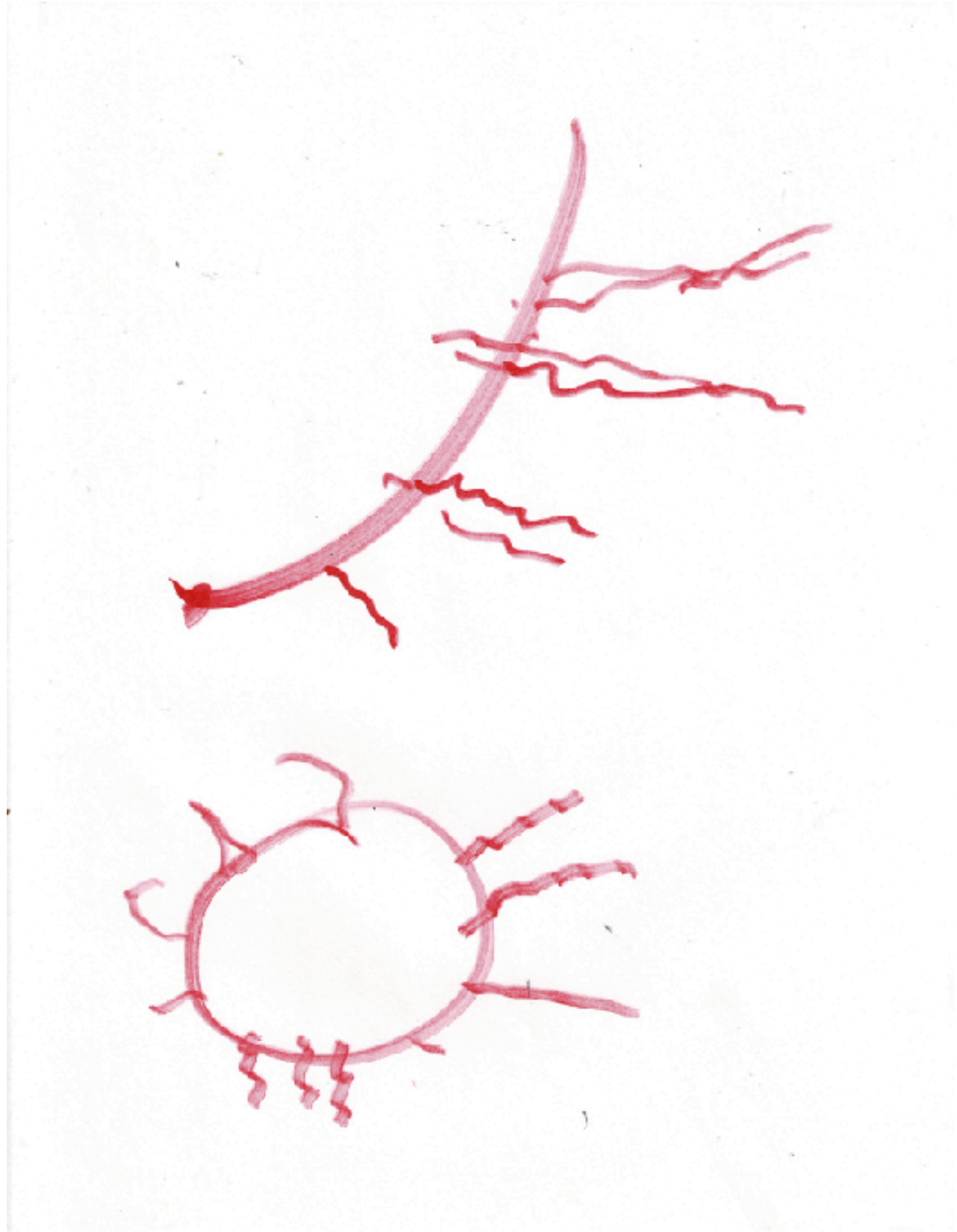
k	10^k	A377906 @ n =	A377904	A377908
0	1	1	1	0
1	10	12	14	2
2	100	16	20	4
3	1000	56	97	41
4	10000	93	176	83
5	100000	136	396	260
6	10^6	168	463	295
7	10^7	321	1918	1597
8	10^8	332	1984	1652
9	10^9	363	2278	1915
10	10^{10}	409	?	?
11	10^{11}	411		
12	10^{12}	443		
13	10^{13}	467		
14	10^{14}	1658		
15	10^{15}	1688		
16	10^{16}	1699		
17	10^{17}	1708		
18	10^{18}	1715		
19	10^{19}	1720		
20	10^{20}	?		



If we ignore the “solar flares”, what is the equation to the principal line?

Sequence A302656 (“S”) is hard to analyse because of the enormous “solar flares” at random-looking (see A377904) intervals.

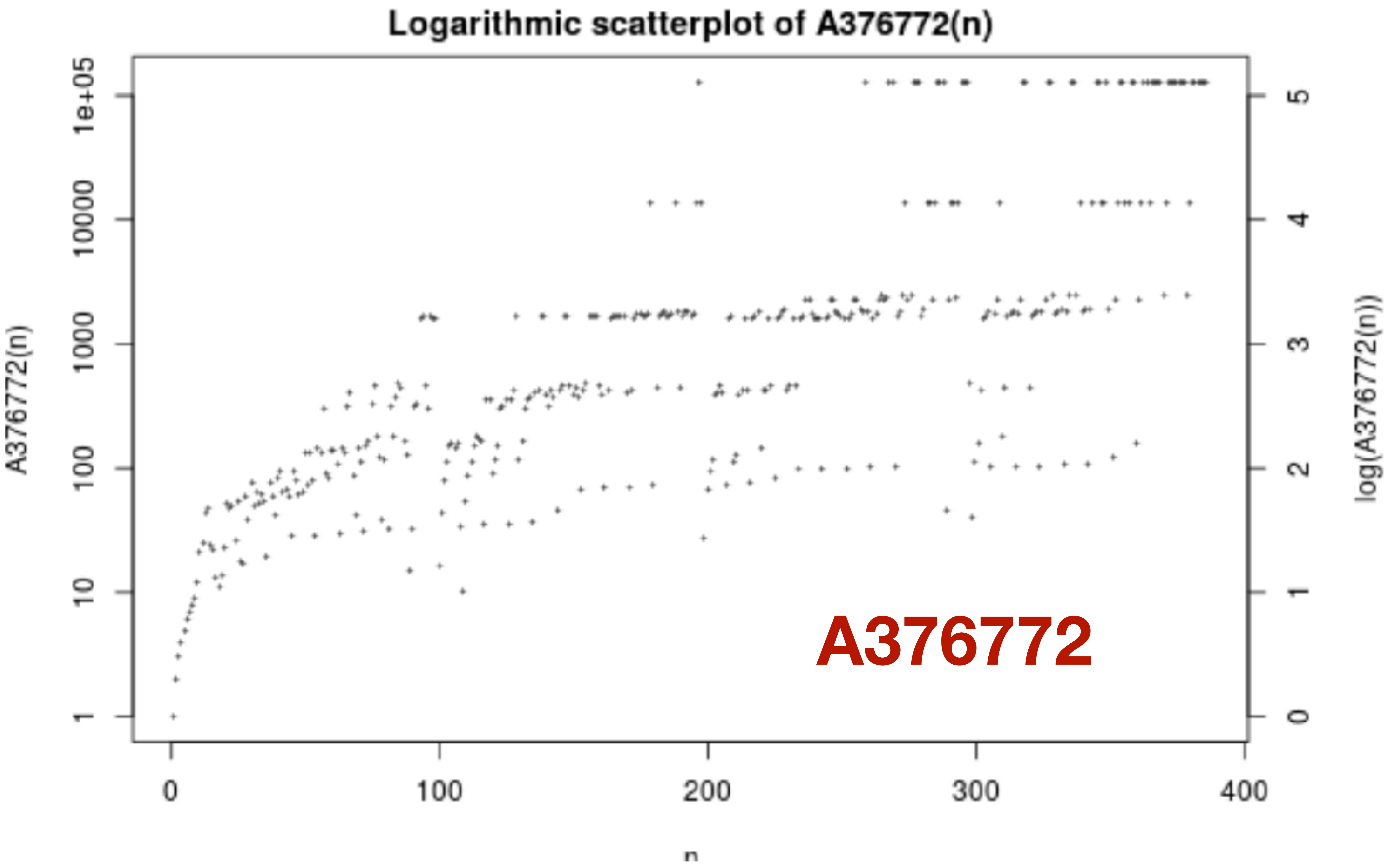
There are 16 derived sequences, A376769 - A376776 and A377903 - A377911, none related to any other OEIS entry (so far!)



Is there a variation of A302656 where the points (n, a(n)) lie on a parabola $y = c x^2$ except for some “solar” flares?

Or on a circle?

Open Problem: Show that A302656 contains every positive integer.



A376772

Where n appears in A302656,
or -1 if n is missing.
Computed by Dominic McCarty:
387 = 9.43 is still missing after
1262743 terms.
Still missing after 3 million terms.

See also A376776:
where prime(n) appears in A302656,
or -1 if it is missing.

“OULIPO”, a strong influence on E.A.

“Ouvroir de Littérature Potentielle” (Workshop for Potential Literature}
- impose artificial constraints, explore the consequences.

“Constraints Breed Creativity”

Raymond Queneau and François Le Lionnais (1960).
Example 1: Georges Perec, “La Disparation” (“A Void”) Novel without letter e, 1964,
e = “eux” = “them” in French, are missing
(his parents died in World War II)

Example 2: Okapi style: vowels and consonants alternate (see later!)

“Even Digit, Next Bigger” E.A., Feb. 2021

“In the digit stream, if a digit is even, the next digit is bigger”
(a classic “Oulipo”-type constraint). **A342042**

Expanded definition: Lexicographically Earliest infinite Sequence of distinct nonnegative integers such that if a digit d in the digit stream (Ignoring commas) is even, the next digit is > d.

A342042

n	1	2	3	4	5	6	7	8	9	10	11	12
a(n)	0	1	2	3	4	5	6	7	8	9	10	11
n	13	14	15	16	17	18	19	20	21	22	23	24
a(n)	12	30	13	14	50	15	16	70	17	18	90	19
n	25	26	27	28	29	30	31	32	33	34	35	36
a(n)	23	24	51	25	26	71	27	28	91	29	31	32
n	37	38	39	40	41	42	43	44	45	46	47	48
a(n)	33	34	52	35	36	72	37	38	92	39	45	46
n	49	50	51	52	53	54	55	56	57	58	59	60
a(n)	73	47	48	93	49	53	54	55	56	74	57	58

Credits for this section :
Michael Branicky,
Sebastian Karlsson,
Kevin Ryde,
Rémy Sigrist,
NJAS,
Paolo Xausa

Even Digit, Next Bigger (cont.)

What numbers appear in S = A342042? Let P = permitted numbers = n such that
every even digit d, except the last, is followed by a larger digit:
A377012 = 0, 1, 2, … 19, 23, 24, … 39, 45, … 59, 67, …
Clearly n in S implies n in P.

Theorem (Sebastian Karlsson, 2021 : Every number in P is in S (so S is a permutation of P)

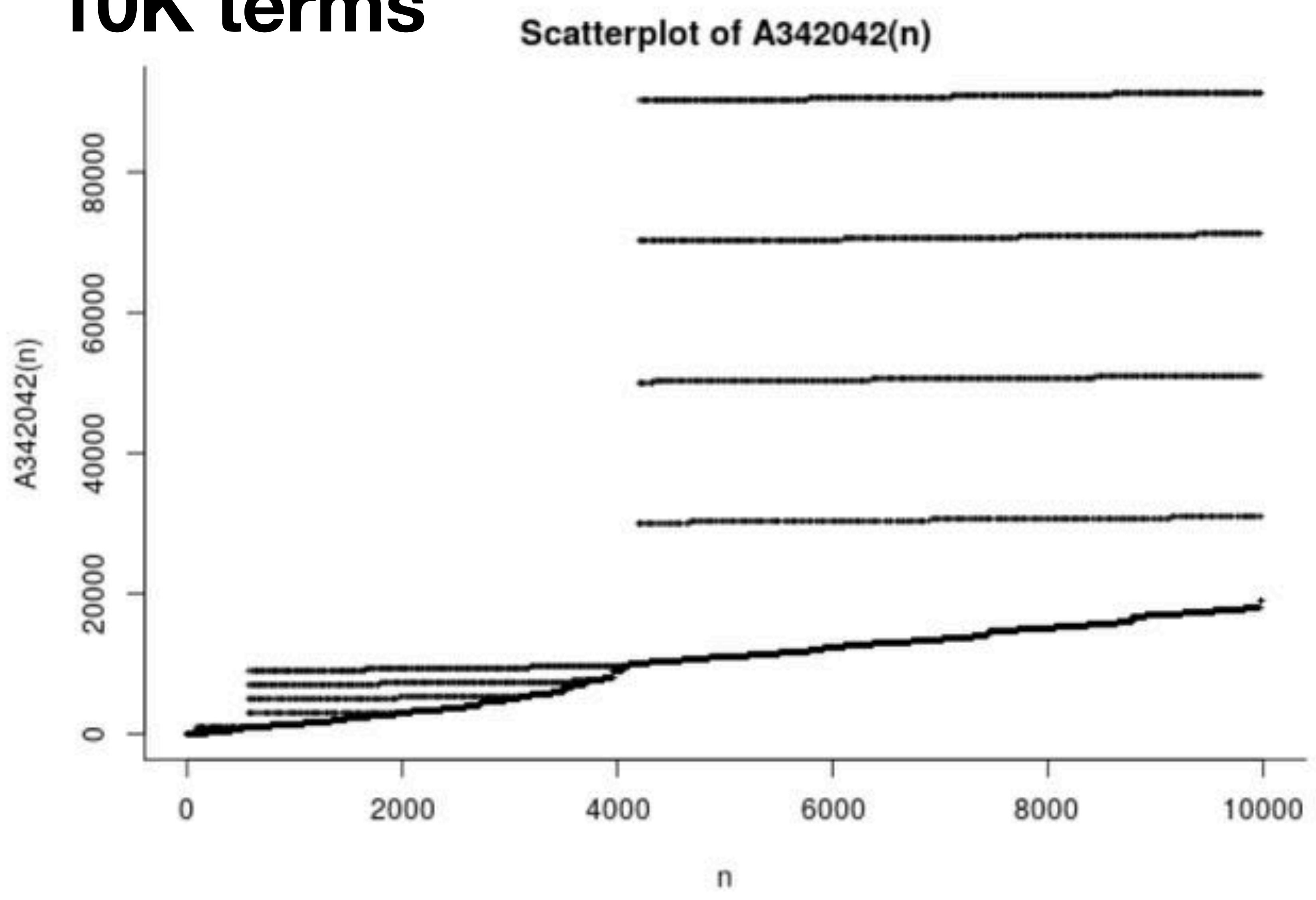
Lemma: Given S = {a(n)}, a sequence of distinct nonnegative numbers.
Let w(n) = when n appears, or -1 if n is missing; let W(n) = max{w(k), k <= n}.
Then i > W(n) implied a(i) > n.

Proof of Theorem: Let x = smallest number in P that is not in S.
By definition, if a(n) is odd, a(n+1) = smallest number in P missing from S.
If infinitely many odd terms in S, choose odd a(n) > W(x).
Then a(n+1) = x, contradiction.

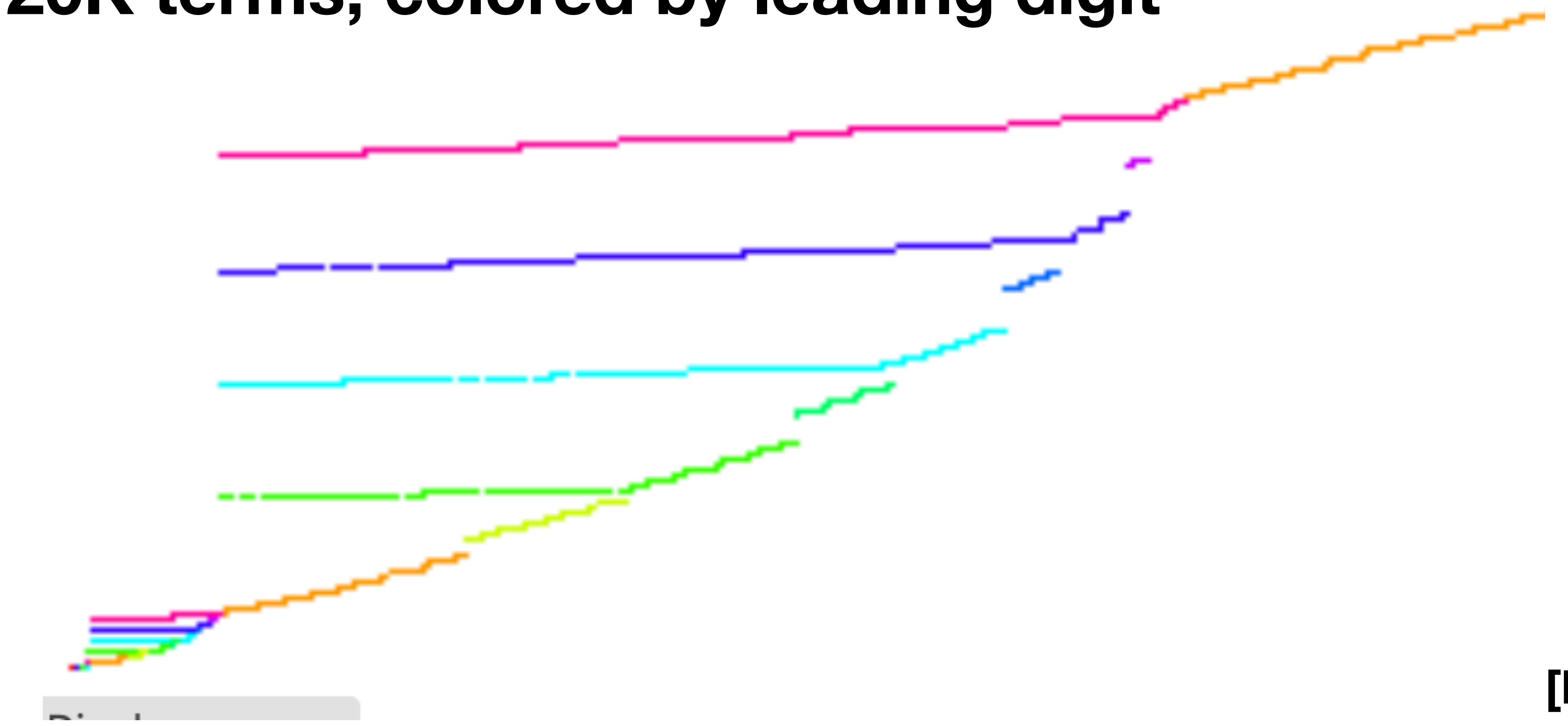
If only finitely many odd terms in S, there are infinitely many missing odd numbers beginning with 9.
Let y = smallest, and n = W(y)+1. Then a(n) > y, so y would have been a smaller choice for a(n).
Contradiction. QED

The Remarkable Graph of Éric Angelini’s A342042

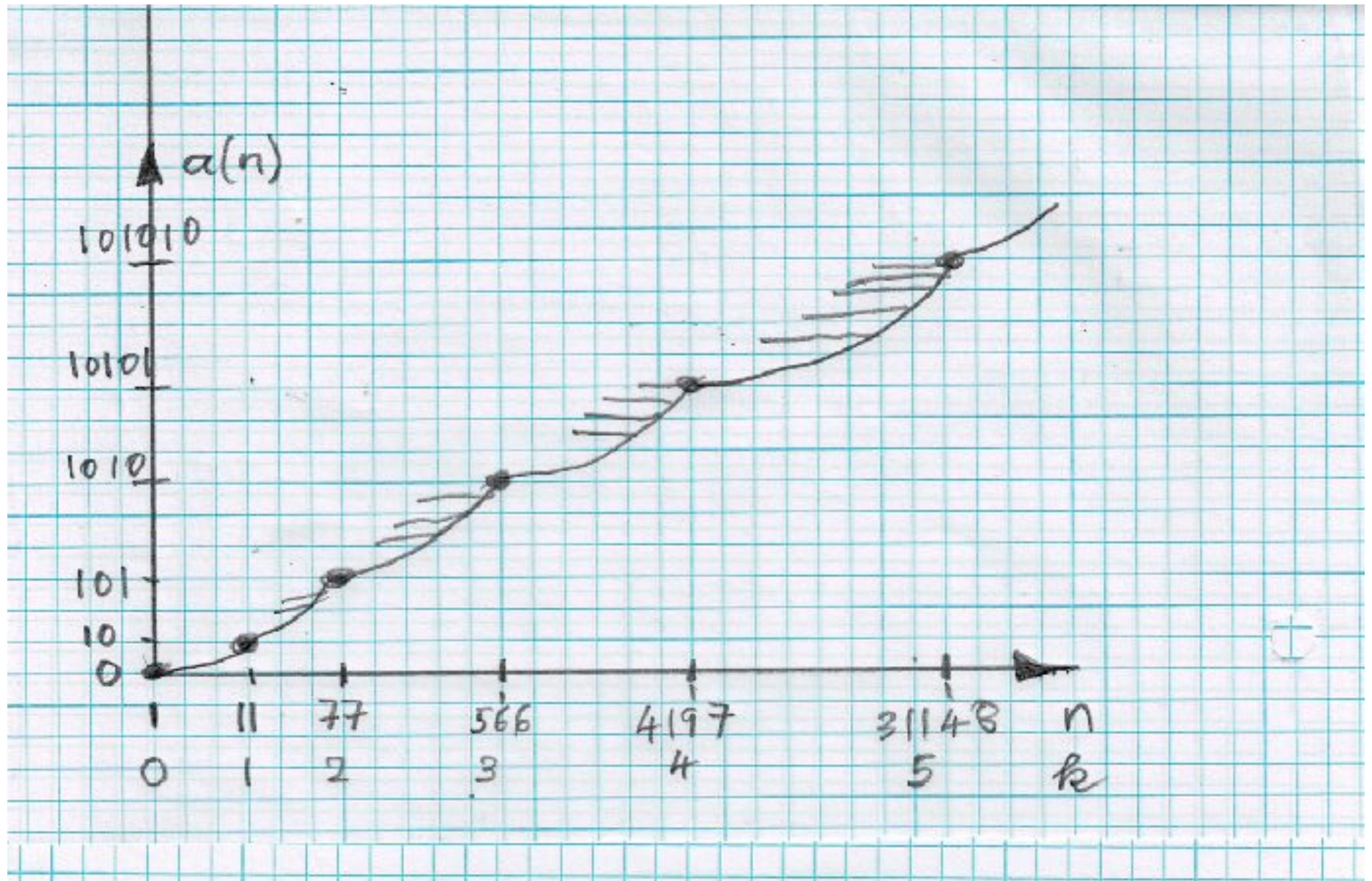
10K terms



20K terms, colored by leading digit



[Rémy Sigrist]



The “nodes” • have coordinates

$$(x_k, y_k) \text{ where } x_k = c_2 c_1^k, y_k = 10^k$$

which (see next slide) lie on the curve

$$y = x^{1.14869...}$$

The Remarkable Graph of Éric Angelini's A342042 (cont.)

The "nodes" are when we come to the end of the numbers in P of lengths k = 1, 2, 3, …
There are 10 of length 1, 66 of length 2, 489 of length 3, etc., see A377917.
Kevin Ryde pointed out that these are the words in a regular language.
He and Michael Branicky found a g.f. for this sequence, which has denominator (x+1)^6 - (x+2). Smallest root is 0.134724…, with reciprocal c_1 = 7.422574.
So nodes in graph have coords

$$(x_k, y_k) \text{ where } \mathrm{x_k = c_2 c_1^k, y_k = 10^k}$$

and c_2 = 1.3824…By eliminating k, we get the curve

$$y = x^{1.14869\ldots}$$

where the exponent is log(10)/log(c1)

This is a rough approximation to the original sequence A342042.

(Proof uses fact, established by Sebastian Karlsson, Helsinki University., that all numbers of the same length appear as a consecutive block.)

In A342042, all numbers with the same number of digits appear together in a block.

“Look Left” and Say What You See

Recall the famous “Say What You See” sequence A5150
1, 11, 21, 1211, 111221, 312211, …

Eric Angelini, Blog, Cinquante Signes, Nov. 2019; **A329447** with Maximillian Hasler.
Sequence begins 0, 10, 11, 20, 12, …
To get next term: Look left and do “say what you see” for ALL digits to the left:
3 0’s, 4 1’s, 2 2’s, that is, write down 30, 41, 22 AND PICK THE SMALLEST (22) -
that is a(n):

0, 10, 11, 20, 12, 22, 30, 13, 23, 33, 40, 14, 24, 34, 44, 50, 15, 25, 35, 45, …

After sorting (**A376779**):

0, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, …

It should be easy to characterize these numbers.

A329447 = minimum value of cd

A377905

Table gives values of c

A329447 ↓	0	1	2	3	4	5	6 ··· value of d
0	1						
10	2	1					
11	2	3					
20	3	3	1				
12	3	4	2				
22	3	4	4				
30	4	4	4	1			
13	4	5	4	2			
23	4	5	5	3			
33	4	5	5	5			
40	5	5	5	5	1		
14	5	6	5	5	2		
24	5	6	6	5	3		
34	5	6	6	6	4		
44	5	6	6	6	6		
50	6	6	6	6	6	1	

This is an LES sequence.

At step n, out of all the true statements "cd" meaning there are c copies of d to the left, pick the smallest

E.A.: A puzzle:

1 2 4 8 16 32 64 128 256 512

1024 2048 4096 8192 16384

65536

32768 3 6 12 24 48 96 192

384 768 1536 3072 61 1 2 4 8

Periodic, easy - explain!

A320487

Eric Angelini's remove-repeated-digits operation

Drop any digit from n that appears more than once

1231 becomes 23, likewise for 1123, 123111, 11023 etc.

Write 0 if nothing left.

In one step, n becomes A320486(n):
1, 2, 3,…,10, 0, 12, 13,…, 21, 0, 23,…

Get 0 with probability 1, so easy to analyze!

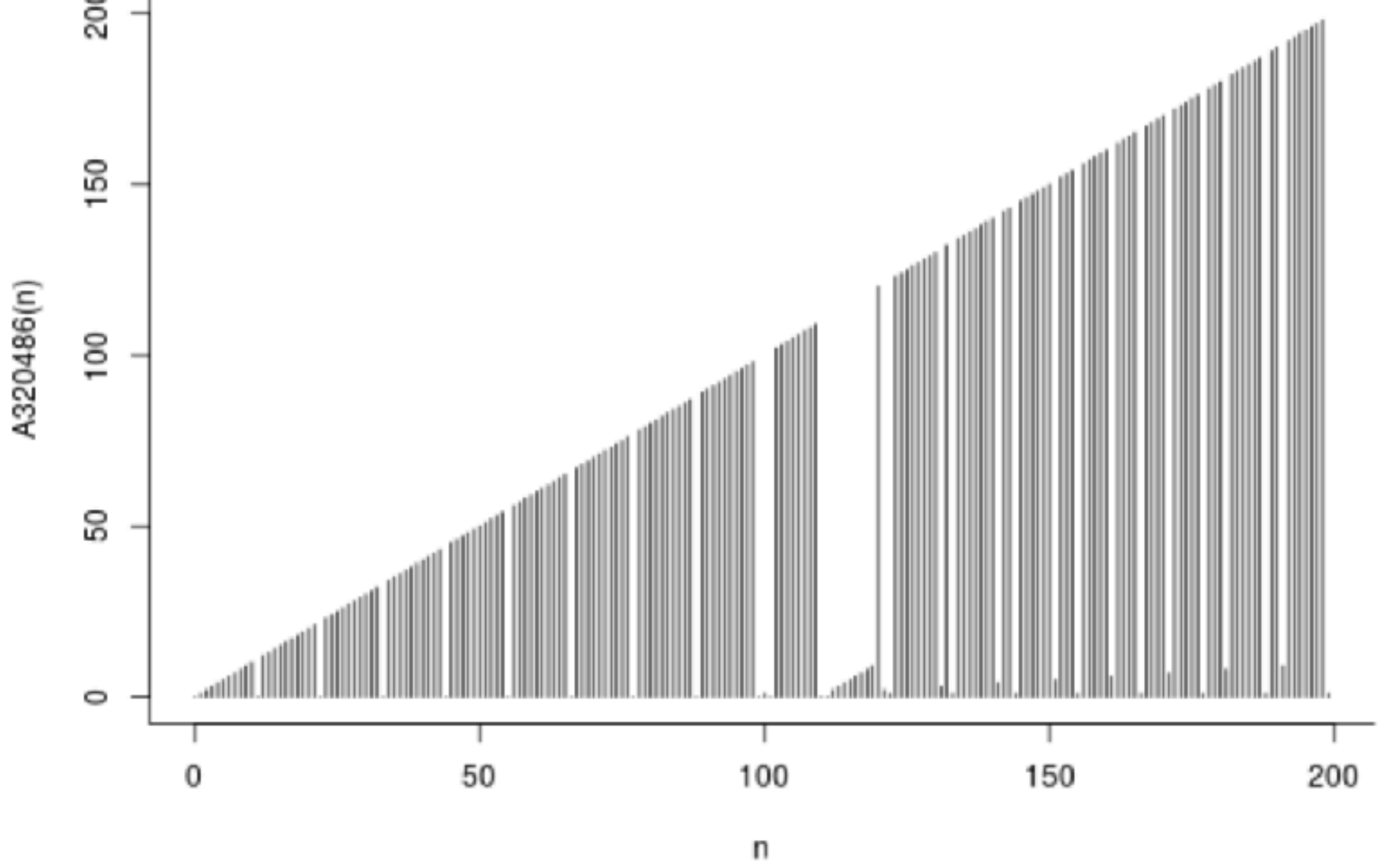
"Factorials"   1, 2, 6, 24, 120, 720, (5040) 54, 432, (3888) 3, 30, (330) 0

A321008

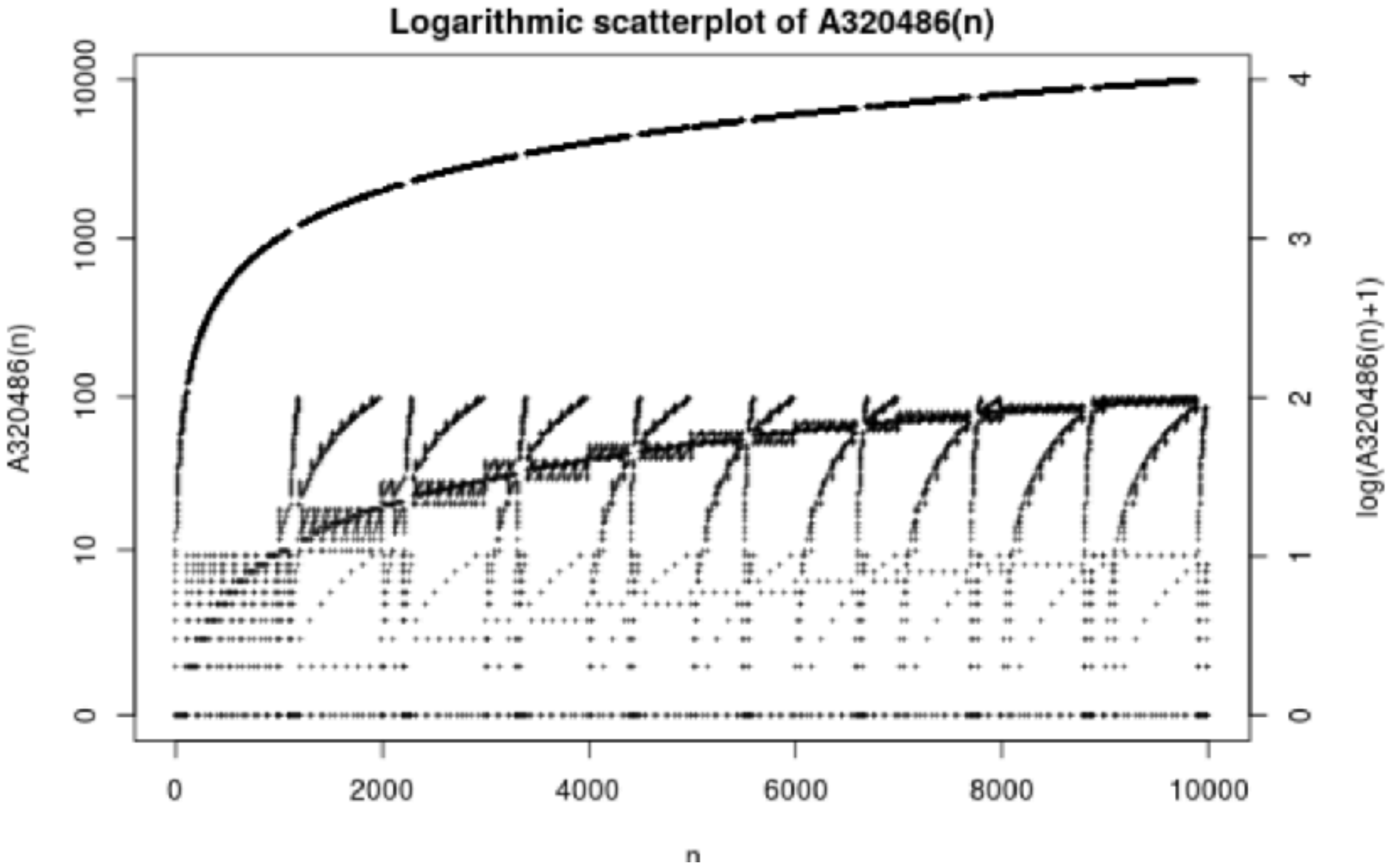
Start with n, and repeatedly square-and-delete:
Conjecture (Lars Blomberg) : Reach one of 5 fixed points:
0, 1, 1465, 4376, 89476.   (A321010)
or one of two nontrivial loops

(1465 is a fixed point: 1465^2 = 2146225 -> 1465)

Two plots of A320486, Angelini's Remove repeated digits from n



200 terms



Log plot of 10K terms

The Rigidity of the Okapi

[Raul654, Disney's Animal Kingdom, 01/16/2005]

In Oulipo, Okapi-style (meaning "striped") might mean vowels and consonants must alternate:

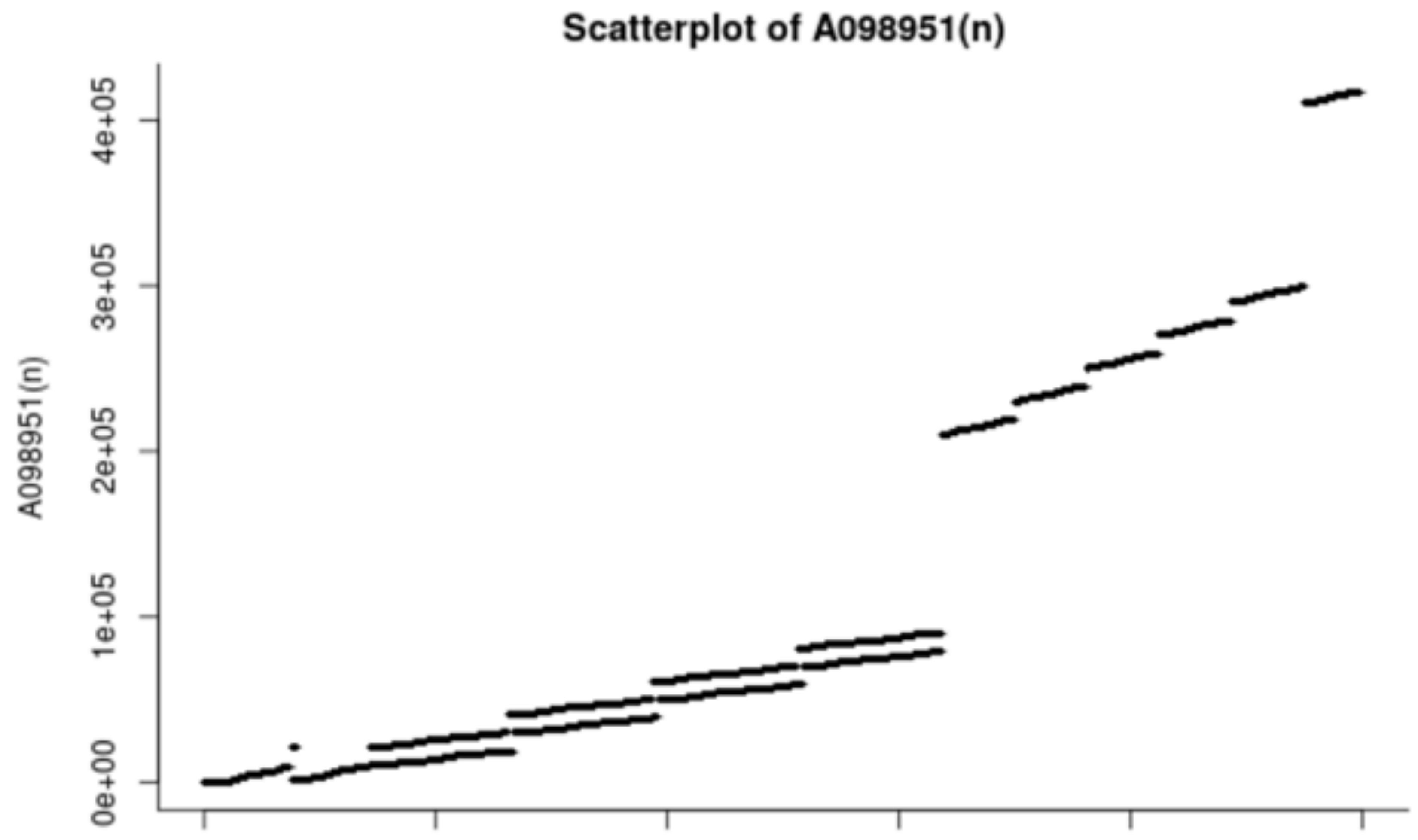
**"Any banana can open a safe,
but a Japanese Sumo tulip is unab[l]e to."**

E.A. (2004): Okapi Sequence 1: Digits must alternate in parity, always pick smallest missing legal number:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 21, 23, 25, 27, 29, 41, 43, 45, 47, 49, 61, 63, 65, 67, 69, 81, 83, 85, 87, 89, 210, 10, 12, …

A098951 (the sequence)
A030141 (the legal numbers)

(LES sequence if compare terms numerically)



NJAS, December 2024 (hommage à Éric Angelini) : Okapi Sequence 2:
LES such that digits alternate in parity, always choose smallest,
comparing terms lexicographically
(as decimal strings).

0, 1, 2, 10, 101, 21, 210, 1010, 10101, 2101, 21010, 101010, 1010101, 210101, 2101010, 10101010, 101010101, 21010101, 210101010, 1010101010, 10101010101, ...

A377919

Arrange the nonnegative integers whose digits alternate in parity in lexicographic order:
L = [0; then the numbers with first digit 1: 1, 10, 101, 1010, 10101, 101010, ...;
then the numbers with first digit 2: 2, 20, 201, 2010, 20101, 201010, ...;
then the numbers with first digit 3, and so on]

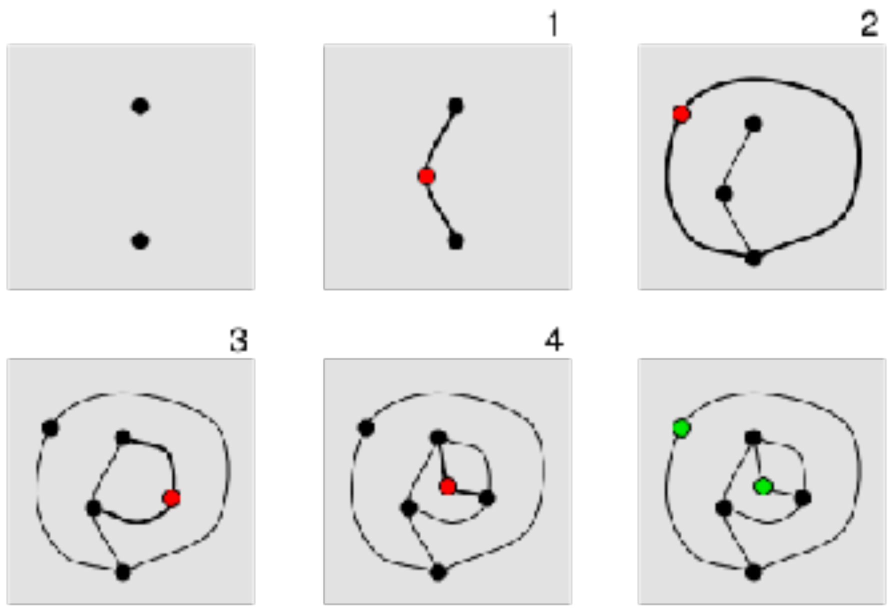
For the sequence, start with 0, extend by adding first unused number from L that preserves alternating parity. There is a simple recurrence.

Note: The list L itself is not in OEIS: for example,
there are uncountably many terms between 1 and 10103,
e.g. 10101010…010301.

“Choix de Bruxelles”:
A New Operation on Positive Integers

Eric Angelini, Lars Blomberg, Charlie Neder, Remy Sigrist, NJAS,

Fibonacci Quart., 17 (2019), 195-200; arXiv 1902.01444; Numberphile video August 2020



Game of sprouts

Choux de Bruxelles

Eric Angelini (Bruxelles)

Choix de Bruxelles (2)

A new operation on numbers

20218

Can double any subnumber, or halve it if it is even

20218 goes to your choice of

10218	20428
40218	2029 *
20118	20236
20418	10118
20228	40418
20214	20109
202116 *	20436
	40428
	10109
	40436

If a goes to b then also b goes to a

16 goes to any of

16, 26, 13, 112, 8, 32

Choix de Bruxelles (4)

1 — 2 — 4 — 8 — 16 — any of { 13, 26, 32, 112 }

Going from 1 to 3 takes 11 steps:

1 2 4 8 16 112 56 28 14 12 6 3 **(Lorenzo Angelini)**

Can get from 1 to any number <= 99 (not ending in 0 or 5) in at most 12 steps.

Theorem 1: The connection graph has two components: numbers ending in 0 or 5, and all the rest.

A323454 = $\tau(n)$ = number of steps to reach n from 1 (or -1 if can't)

0, 1, 11, 2, -1, 10, 9, 3, 9, -1, 10, 9, 5, 8, -1, 4, 7, 8, 8, -1, 10, …

Theorem 2: For n not ending in 0 or 5,

$$\log_{10} n + 5 < \tau(n) \leq 12 \log_{10}(n)$$

Choix de Bruxelles (5)

Theorem 3: Starting at n, the biggest number M you can reach in one step is:

if n = 3141592654

find right-most digit >= 5 and double starting there: 54 -> 108

and we get M = 31415926108

Theorem 4: Starting at n, all number M you can reach in one step satisfy

$$\frac{n}{10} < M < 10\,n$$

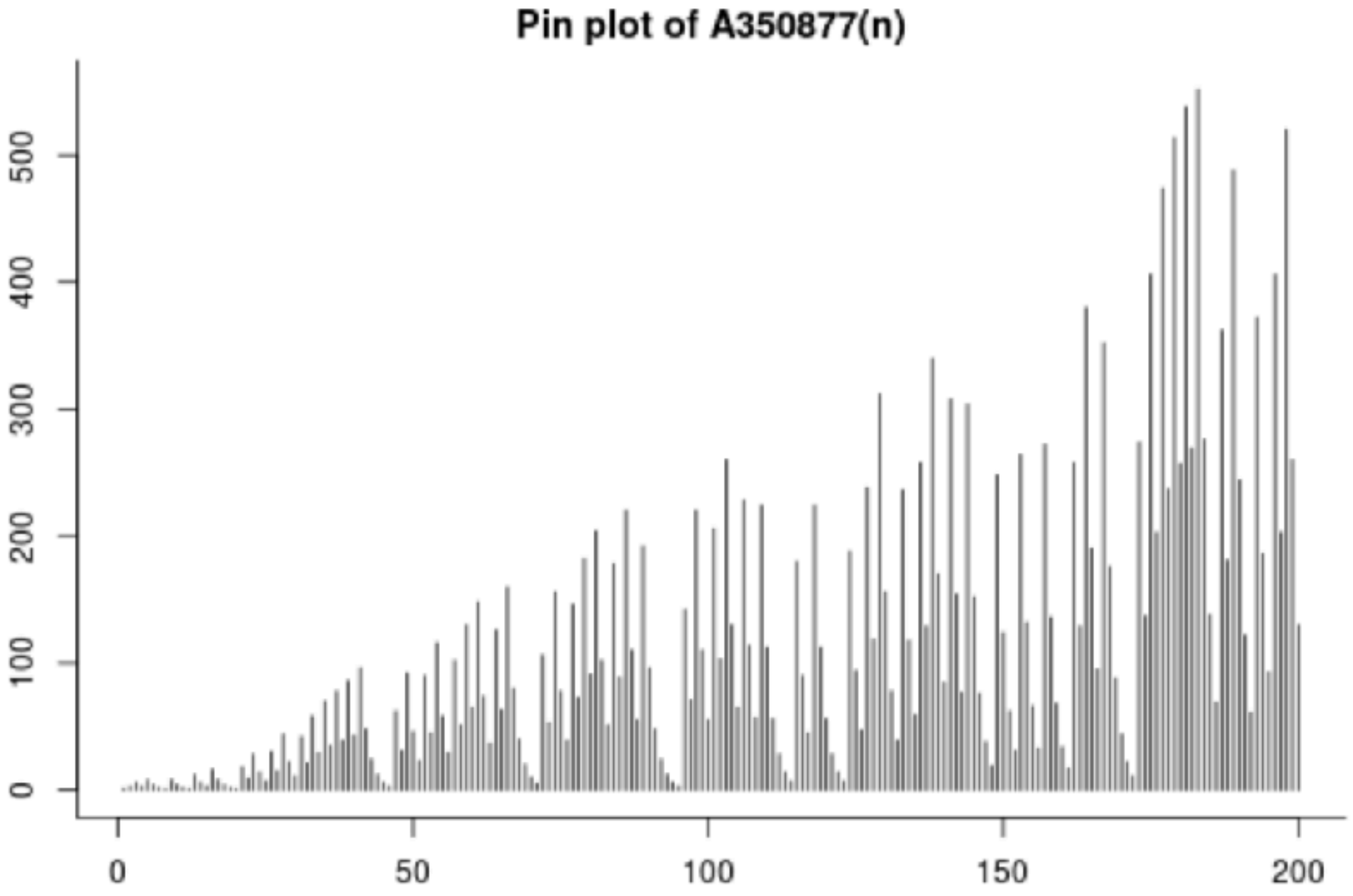
Theorem 5: Starting at 1, the max number M you can reach in $n \geq 14$ steps satisfies

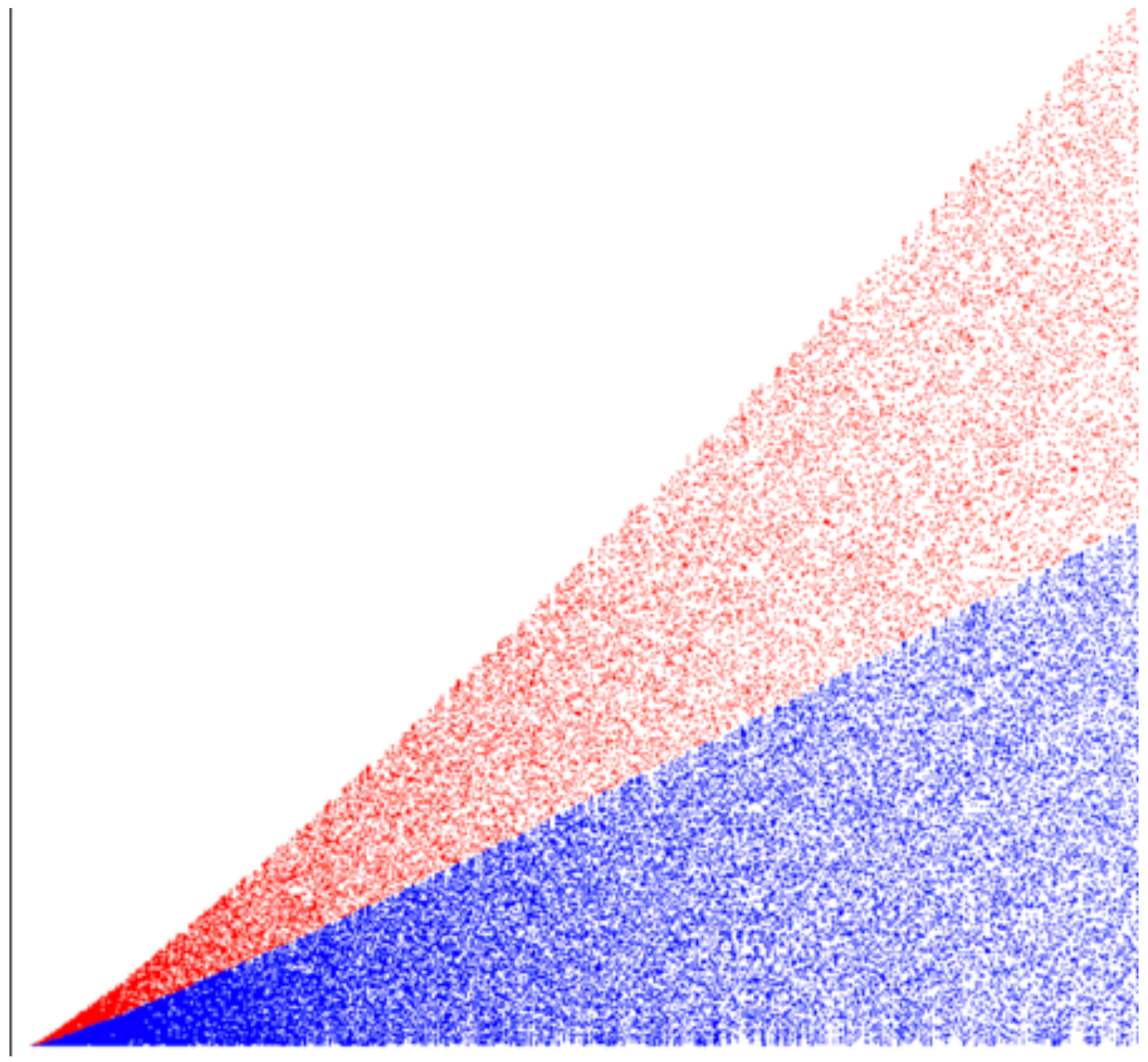
$$8.112\;10^{n-6} < M < 8.113\;10^{n-6}$$

The Sisyphus Sequence A350877, E.A. & Carole Dubois, Jan. 2022

a(1)=1; if even, divide by 2, if odd add next prime

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
a(n)	1	3	6	3	8	4	2	1	8	4	2	1	12	6	3	16	8	4	2	1	18	9	28	14	7	30	15
	2	3		5				7				11			13					17		19			23		





30K terms, slope of upper line approx 7,
red = terms following an odd term

Russ Cox, Michael De Vlieger, Martin Ehrenstein,
Hans Havermann, Rémy Sigrist, Allan C. Wechsler, NJAS and others

The big open question: does every number appear?

After 10^9 terms we were missing 36, 72, … However:

36 is part of a descending chain that ends with a(77534485879) = 9 and starts with a(77534485842) = 1236950581248 = 2^37 * 9, after adding the prime 677121348413 = prime(25844737276).

a(17282073747557) = 97 ends a descending chain that starts with a(17282073747516) = 213305255788544 = 2^41 * 97 after adding the prime 183236837077571. [Martin Ehrenstein]

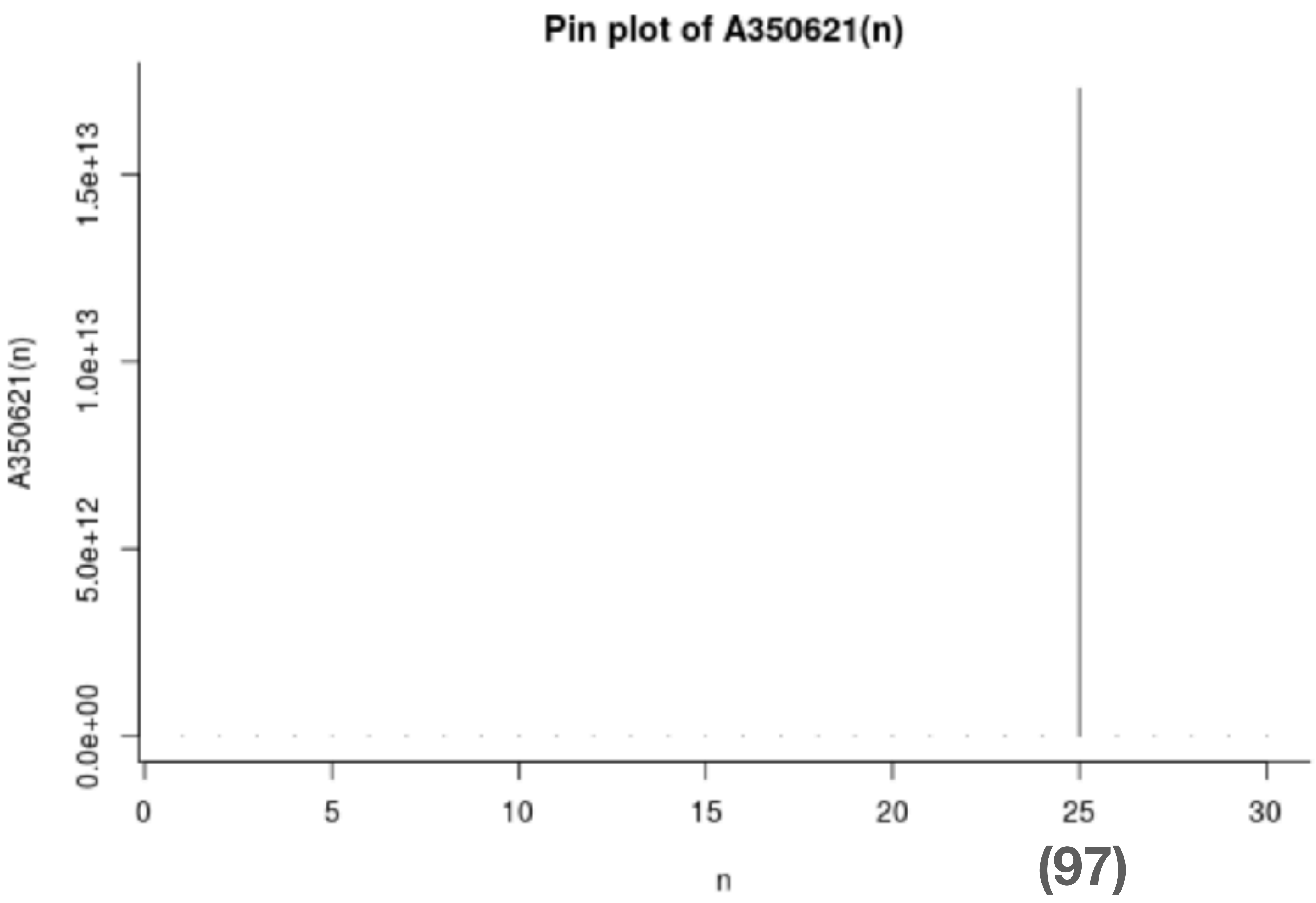
Conjecture: On naive probabilistic grounds, all integers should eventually appear. An up-step is always immediately followed by a down-step, and then, on average, by one more down-step. So we expect that every third step will be an up-step, by the next prime number, which will be around p(n/3). So the sequence will spend a lot of its time between p(n/3)/3 and 4p(n/3)/3.

[Allan C. Wechsler]

A350877 (Sisyphus), continued

When n-th prime appears for the first time, or -1 if it never appears.

7, 2, 71, 25, 30, 345, 161, 148, 51, 34, 48, 63, 234,
40, 126, 73, 135, 192, 454, 97, 78, 24841, 433, 85,
1728207374 7557, 322, 102, 106544217, 207, 556, (?), …



A350621

Martin Ehrenstein

Prime(31) = 127 is stll missing (Dec. 2024)

The Comma Sequence (Eric Angelini, 2006)

Comma Numbers = First Differences

a(n): 1 , 12 , 35 , 94 , 135 , 186 , 248 , 331 , 344 , … A121805

cn(n): 11 , 23 , 59 , 41 , 51 , 62 , 83 , 13 , … A366487

comma numbers

Edwin Clark: a(2137453) = 99999945

and the next term does not exist!

But if we start with 3 we get 3 , 36 (Just 2 terms)

33

If start with 1 2 3 4 5 6 7

the comma sequence has length

2137453, 194697747222394, 2, 199900, 19706, 209534289952018960, 15, …

A330128

The Comma Sequence: For further information see

E. Angelini, M. S. Branicky, G. Resta, NJAS, and D. W. Wilson,
The Comma Sequence: A Simple Sequence with Bizarre Properties,
Fibonacci Quart., 62 (2024), 215-232; arXiv:2401:14346.

Lorenzo Angelini,
Happy
birthday Éric!!, Youtube video.

NJAS, Exp. Math. Seminar, Rutgers, Jan. 2024,
Youtube.

R. Dougherty-Bliss and N. Ter-Saakov, The Comma Sequence is Finite
in Other Bases, arXiv:2408.03434.

A last message?