

# RUTGERS EXPERIMENTAL MATHEMATICS SEMINAR

**Title:** Sharing Pizza in  $n$  Dimensions

**Speaker:** Richard Ehrenborg, University of Kentucky

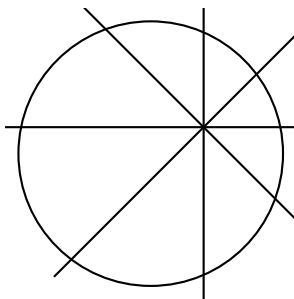
**Date:** Thursday, March 31st, 2022

**Time:** 5:00pm–5:48pm

**Place:** zoom

**Short abstract:** We introduce and prove the  $n$ -dimensional Pizza Theorem. This is joint work with Sophie Morel and Margaret Readdy.

**Long abstract:** We introduce and prove the  $n$ -dimensional Pizza Theorem. Let  $\mathcal{H}$  be a real  $n$ -dimensional hyperplane arrangement. If  $K$  is a convex set of finite volume, the *pizza quantity* of  $K$  is the alternating sum of the volumes of the regions obtained by intersecting  $K$  with the arrangement  $\mathcal{H}$ . We prove that if  $\mathcal{H}$  is a Coxeter arrangement different from  $A_1^n$  such that the group of isometries  $W$  generated by the reflections in the hyperplanes of  $\mathcal{H}$  contains the negative of the identity map, and if  $K$  is a translate of a convex set that is stable under  $W$  and contains the origin, then the pizza quantity of  $K$  is equal to zero. Our main tool is an induction formula for the pizza quantity involving a subarrangement of the restricted arrangement on hyperplanes of  $\mathcal{H}$  that we call the *even restricted arrangement*. We get stronger results in the case of balls. We prove that the pizza quantity of a ball containing the origin vanishes for a Coxeter arrangement  $\mathcal{H}$  with  $|\mathcal{H}| - n$  an even positive integer. This is joint work with Sophie Morel and Margaret Readdy.



<https://sites.math.rutgers.edu/~zeilberg/expmath/>