Title: Sharing Pizza in \( n \) Dimensions

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Time: 5:00pm–5:48pm

Place: zoom

Short abstract: We introduce and prove the \( n \)-dimensional Pizza Theorem. This is joint work with Sophie Morel and Margaret Readdy.

Long abstract: We introduce and prove the \( n \)-dimensional Pizza Theorem. Let \( \mathcal{H} \) be a real \( n \)-dimensional hyperplane arrangement. If \( K \) is a convex set of finite volume, the pizza quantity of \( K \) is the alternating sum of the volumes of the regions obtained by intersecting \( K \) with the arrangement \( \mathcal{H} \). We prove that if \( \mathcal{H} \) is a Coxeter arrangement different from \( A_n \) such that the group of isometries \( W \) generated by the reflections in the hyperplanes of \( \mathcal{H} \) contains the negative of the identity map, and if \( K \) is a translate of a convex set that is stable under \( W \) and contains the origin, then the pizza quantity of \( K \) is equal to zero. Our main tool is an induction formula for the pizza quantity involving a subarrangement of the restricted arrangement on hyperplanes of \( \mathcal{H} \) that we call the even restricted arrangement. We get stronger results in the case of balls. We prove that the pizza quantity of a ball containing the origin vanishes for a Coxeter arrangement \( \mathcal{H} \) with \( |\mathcal{H}| - n \) an even positive integer. This is joint work with Sophie Morel and Margaret Readdy.

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