

The q, t -Catalan Sequence

$$C_n(q, t) = \sum_{\mu \vdash n} \text{a certain rational function in } q, t$$

Example

$$C_2(q, t) = \frac{t^2}{t - q} + \frac{q^2}{q - t} = t + q$$

$$\begin{aligned} C_3(q, t) &= \frac{q^6}{q^2 - t} + \frac{t^2 q^2 (1 + q + t)}{(q - t^2)(t - q^2)} + \frac{t^6}{t^2 - q} \\ &= q^3 + q^2 t + q t + q t^2 + t^3. \end{aligned}$$

$$\sum_{\substack{\nu \subseteq \mu \\ m-d \leq |\nu| \leq m}} c_{\mu, \nu}^{g \perp} T_{\nu} = T_{\mu} G [D_{\mu}(1/q, 1/t)] \text{ and}$$

$$\sum_{\substack{\nu \subseteq \mu \\ m-d \leq |\nu| \leq m}} c_{\mu, \nu}^{g \perp} = F [D_{\mu}(q, t)],$$

where

$$G [X] = \omega \nabla \left(g \left[\frac{X+1}{(1-1/q)(1-1/t)} \right] \right),$$

$$F [X] = \nabla^{-1} \left((\omega g) \left[\frac{X-\epsilon}{M} \right] \right),$$

and

$$D_{\mu}(q, t) = MB_{\mu}(q, t) - 1.$$