Commutation and Rearrangements

An electronic reedition of the monograph

Problèmes combinatoires de commutation et réarrangements

by P. Cartier, D. Foata

with three new appendices by

D. Foata, B. Lass and Ch. Krattenthaler

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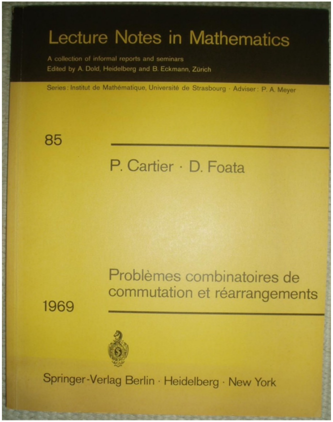

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Actes 18^e Séminaire Lotharingien, p. 53–76

THE CONCEPT OF BAILEY CHAINS

BY

PETER PAULE

0. Introduction

In his expository lectures on q-series [3] G. E. Andrews devotes a whole chapter to Bailey's Lemma (Th. 2.1, 3.1) and discusses some of its numerous possible applications in terms of the "Bailey chain" concept. This name was introduced by G. E. Andrews [2] to describe the iterative nature of Bailey's Lemma, which was not observed by W. N. Bailey himself.

Remark. In proving equation (30) P. Cartier and D. Foata gave a purely combinatorial proof of

$$\binom{b+c+\rho}{c+k}\binom{c+a+\rho}{a+k}\binom{a+b+\rho}{b+k} = \sum_n \frac{(a+b+c+n-\rho)!}{(a-n)!(b-n)!(c-n)!(n+k)!(n-k+\rho)!}, \tag{33}$$

where $\rho = 0$ or 1 (a *Pfaff-Saalschütz* summation).

From that the proof of (29), the extension of Bailey's Lemma in the cases $N = 0$ or 1, for $q = 1$ is simple: one just has to multiply equation (32) by coefficients c_k and to sum over all possible k's and finally change the order of summation.

Recently [31] D. Zeilberger q-analogized the Cartier-Foata proof of (33) to show

$$\begin{bmatrix} b+c \\ c+k \end{bmatrix} \begin{bmatrix} c+a \\ a+k \end{bmatrix} \begin{bmatrix} a+b \\ b+k \end{bmatrix} = \tag{34}$$

$$= \sum_n q^{n^2-k^2} \frac{[a+b+c-n]!}{[a-n]![b-n]![c-n]![n+k]![n-k]!}.$$

This gives equation (29) with $\rho = 0$ as described above.

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A q-Foata Proof of the q-Saalschütz Identity

Doron Zeilberger

Lotharingien conference dinners

Banquet": painting by Ingrid Dohm

Lotharingien waltz dancing (SLC 42, 1998, Maratea; Andrews-Fest)

https://www.britannica.com/art/waltz#/media/1/635186/7966

Dominique Foata: the pioneer hero of Lotharingien Combinatorics

Taken from: **"Richard Wagner's The Ring of the Nibelung"** by P. Craig Russel