

# Dick Askey (1933-2019) and what I've learned about him and his life

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Theoretical Computer Science**

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- 8 Askey at the University of Chicago (1961-1963)
- 9 Askey at the University of Wisconsin-Madison (1963-2019)

## Sources for this talk on Dick Askey

- Robert Lange's *Oral History Interview* at UW-Madison with Richard Askey on October 23, 24, 25, 2006. Total time = 413 minutes. Transcript editing "complete". Transcription paid for by George Andrews, Mourad Ismail and George Gasper.
- *Celebratio Mathematica*: Financially supported by Krishna Alladi, George Andrews, Bruce Berndt, Howard Cohl, Mourad Ismail, Dennis Stanton, Sergei Suslov, Roderick Wong, Doron Zeilberger, and Tom Koornwinder.
- Askey's Oral Interview for the R.L. Moore Collection
- Askey's Ph.D. thesis
- Askey's publications (Tom Koornwinder)
- Askey's university transcripts
- Askey's faculty file (redacted) at UW-Madison
- Askey Liber Amicorum
- Conversations with Dick's colleagues and students
- Plus I've met him on several occasions and had spoken with him.

# Group photo of Askey 80 conference 2013 in Madison, WI



- At a conference, after I had a query about **Rahman's** paper on the  ${}_9F_8$  linearization coefficients of Jacobi polynomials, Askey told me: *Howard, read the paper!*  
**Good advice!**



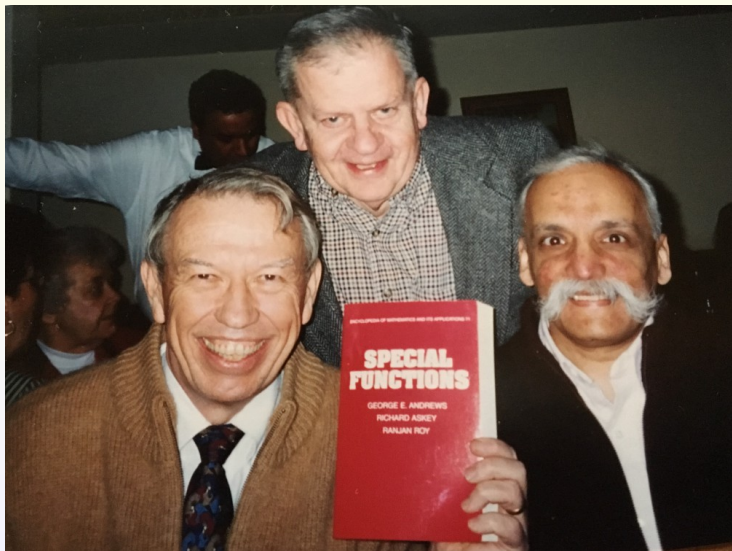
# Some of the many accomplishments of Richard Askey

- The Askey scheme of generalized hypergeometric orthogonal polynomials
- The  $q$ -Askey scheme of basic hypergeometric orthogonal polynomials
- The introduction of the Askey–Wilson polynomials
- Study of the connection between combinatorics and orthogonal polynomials
- The support in the proof of the Bieberbach conjecture
- Andrews–Askey–Roy *Special Functions* (1999), CUP, Cambridge
- *Orthogonal Polynomials and Special Functions* (1975), SIAM, Philadelphia
- Importance of connection coefficients, linearization coefficients, product integral representations, Jacobi series, ...

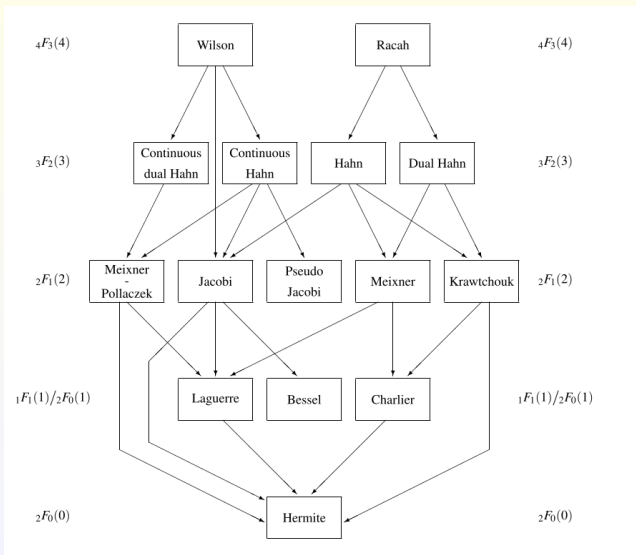
## Andrews–Askey–Roy (1948–2020)

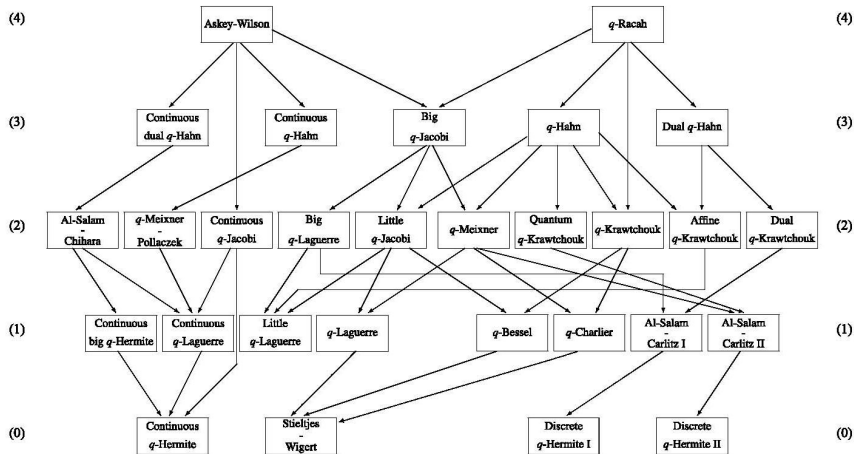
- This is Mourad Ismail's commentary: *For instance, the idea of this book had emerged in the early 70s. Dick had mentioned to me that since the addition theorem for Jacobi polynomials had been complete, it was time to write this up as a book. He indicated that he would like me to review the proofs in the book. However, then Askey discovered the existence of combinatorial problems associated with orthogonal polynomials and realized that there was a lot left to do. Stanton worked in combinatorial problems. Ultimately, the book wasn't published until more than 20 years after he had the original idea.*
- Andrews and Askey have said on several instances, that the publication of this book is mostly due to the hard, persistent and crucial work of Ranjan Roy.

# Andrews–Askey–Roy (1948-2020): *Special Functions*



# The Askey scheme



The  $q$ -Askey scheme

## Some of the many accomplishments of Richard Askey

- Edited Szegő's collected works (w/commentary). **Barry Simon:**

*On behalf of the mathematical community, I should like to thank you for your efforts on putting together Szegő's complete works. Your additions turned what might have been a dry compendium into lively reading and made the document much more valuable. It was clearly a labor of love for you but also immensely valuable for those of us who shared interests with Szegő.*



## Some of the many accomplishments of Richard Askey

- Work on History of Mathematics. Dick helped to edit:  
*A Century of Mathematics in America*.
- Much work on Mathematics Education.
- Famous for knowing the literature.

# Apologies for important topics that will most likely not be covered in this talk

- Some aspects of **Askey's** view on mathematics, the history of mathematics, positivity and grades.
- **Askey's** collaboration with **George Andrews** and Andrews' discovery of the **Ramanujan's** lost notebooks, Askey's visits to India with Andrews, **Berndt** and **Alladi** in connection with Ramanujan and the story of the Ramanujan's busts and Ramanujan's wife.
- Recollections of the proof of the **Bieberbach conjecture**.
- **Askey's** story about the "AAAA" paper.
- Personal reminiscences about **Dick's** 'students', postdocs, colleagues, and collaborators (You know who you are!), and the various conferences that **Dick** attended and lectured at.
- If you're interested in learning more about these, you'll have to scour the literature or wait until the **Askey Springer Volume** is published.



# Askey Springer Volume

- Edited by **George Andrews, Howard Cohl, Mourad Ismail and Luc Vinet**
- Collection of historical vignettes, remembrances and research papers with strong connections to **Richard Askey**
- Received contributions: **Ken Ono, David Bressoud, Nico Temme, Ernie Kalnins, Roderick Wong, Dennis Stanton, Bruce Berndt, Slava Spiridonov.**
- Expected and ongoing contributions: **Cohl, Paul Nevai, Charles Dunkl, Michael Schlosser, Jacob Christansen, Hjalmar Rosengren, Luc Vinet, Mourad Ismail, Wadim Zudilin, Ole Warnaar, Donald Richards, Hung-Hsi Wu, ...**
- Tentative submission deadline: **December 31, 2024.**

## Askey—the early years

- Dick Askey was born in St. Louis, Missouri on June 5, 1933.
- His father had a B.S. in Business Administration.
- His mother had one year of college. She had mathematical ability. She encouraged Askey to learn in school and was told that grades are not important, it was what he learned that was important.
- The only thing that was ever said is my father was a little concerned *where was I going to make a living?* He was never worried about that.
- His parents moved to Baltimore after his eight grade. Askey graduated from Baltimore City College, which is a high school.
- Dick said he was always good at math.

## Askey—the early years

- Dick said he was always good at math.
- When he was about three and still sitting in a high chair, he called his mother and asked her if she could bring him the calendar because he'd finished the clock.
- He was doing a follow-the-dot puzzle and he didn't know the numbers yet, but he knew the numbers came in a sequence and that they were in clocks and calendars.
- When he was in Garfield Elementary School, he remembers walking down the hall and seeing a 5th or 6th grade class (he was probably in 3rd grade). He saw something on the board and didn't understand it at all. It was a square root sign. He asked the teacher about that and she showed how to take square roots. Then in reading a one-volume encyclopedia at home, he read how to take cube roots (based on the binomial expansion).

# Askey dreaming about the sequence of numbers on a clock



## Askey—the early years

- In high school he learned the binomial theorem and figured out how it all worked. He said he's gone back and read school math text books and there used to be beautiful explanations for why the square root algorithm and the cube root algorithm works in textbooks from 1870 through about 1905, 1910 and then it disappeared.
- In 8th grade, they finished his 8th grade arithmetic program with about six weeks to go and the teacher said, *Well, what I'm going to do is I'm going to teach you algebra. Now, you don't have to learn it, but you have to be quiet, and you can do work for other classes, you can bring a book to read, but anybody who wants to learn algebra, I'm going to teach you some algebra.* So he went through a lot of algebra.

## Askey—the early years

- In 9th grade his family moved to Baltimore. In the second semester they bought a house half way through the year he moved from one school to another. Askey: *The teacher regularly gave one advanced problem to the students. The problem that she gave was: You're in a ship with a mast that's a hundred feet high. How far can you see? Well, quite a few of us solved the problem. I remember solving it myself, having found the Pythagorean Theorem somewhere.*
- Askey: *The next year in geometry, at the very beginning I had trouble. It's the only time I had trouble in mathematics in school in my life, until I got to college. Because I didn't understand what it was they were doing and why they were doing it, right at the very beginning about axioms. But, that took a couple of weeks to straighten out.*

## Askey—the early years

- *Askey: Then, the next year, the school I was in had an advanced placement program where you could start in ninth grade in an accelerated program, and your last year you were basically taking college courses. You went through the four years in three.*
- *Askey: Eleventh grade math was a semester of intermediate algebra and then trigonometry, and twelfth grade was solid geometry and then advanced algebra. Solid geometry didn't need either of the other two as a prerequisite, so I took both of them so that I could take analytic geometry and some calculus the senior year with the advanced placement group.*

## Askey—the early years

- *Askey: I ended up in the wrong room for the first class. The two rooms that was supposed to be in were right next to each other, and the teacher I had was a showoff. He had a calculus book on his desk, but I never saw any indication that he really knew any calculus, and he started the advanced algebra class by saying, now there are some equations that you can't solve, and he put on an equation that had complex roots, and he said, we're going to learn how to solve these problems. He said, now none of you know how to solve that. I raised my hand, and I said the answer is such and such. He looked at me, and said, where did you learn that? At least I had enough sense not to say that I learned it in eighth grade. [laughs] I said, oh, I learned it a while ago in another class, but that's how much algebra had been done in that eighth grade class. We'd gotten through quadratic equations and complex roots.*



## Askey—the early years

- *Askey: In the other class, I had a very good teacher, an elderly man with long white hair, who was the head of the math department. He taught in what would now be considered a very old-fashioned way. He would lecture for about five minutes at the beginning of the period, talk about what was happening, and then give a bunch of problems for us to do, and then he'd walk around the room. ... He had enough confidence so that he began to realize that I was fairly good.*
- *Askey: The next year when I took analytic geometry and calculus from him, this was a smaller class because it was an accelerated program. There were about fifteen people in the class. What he did is he split the class. He took about ten of the students and I took five so that he did it the same way. He would lecture for a little while, have problems, but when the students had problems on the homework problems, the class problems, the few of them around me would come to ask me rather than to ask him.*

## Askey—the early years

- *Askey: He told my parents that he never taught me anything, which isn't true; he did teach me a number of things. He probably didn't teach me any mathematics that I didn't know because I had, by that time, started to work ahead on my own, but he taught me something about how to teach and how to work with students.*
- *Askey: I started tutoring students probably in about eighth grade, just in class, answering questions. When I was in high school the honor society set up a tutoring program before school. I was probably the instigator of it, because I had been doing tutoring, just informal tutoring before, and I thought that that was one of the responsibilities that better students had, was to be able to help the other students. That was very valuable in terms of learning how to teach and how to work with students, plus that eleventh grade trigonometry tutoring. So, that's my school education.*

## Askey–Washington University in Saint Louis–admitted

- Askey had been living in Baltimore, Maryland at the time. But he didn't apply to Johns Hopkins because he didn't want to live at home.
- He applied to MIT and Washington University in Saint Louis (Washington University) and was accepted in both places.
- MIT gave him an honorary scholarship (an honorary scholarship means no money) and Askey's family had four children, and he didn't want to ask his parents to pay that much,
- Askey's father had been an undergraduate at Washington University, and he'd grown up in St. Louis. So he enrolled in Washington University in Saint Louis on September 28, 1951.
- The Askey family lived in or near Little Rock, Arkansas from 1951 until 1956 when they moved to Allentown, Pennsylvania. Askey's father took a job managing bus companies in Pennsylvania, Ohio, Delaware.

## Askey family travels, trials, and tribulations

- During Askey's father's tenure as Vice-president, in 1955 there was a violent strike at the Capitol Transit Company over a wage dispute which directly affected, to say the least, Askey's family. Askey's younger brother Phil said: Fuel storage tanks were set afire, our house was shot up (a dozen bullets through Carol's bedroom window).
- This was the *Little Rock Bus Strike* of 1955-1956: On June 22, 1955, 178 drivers and mechanics of Division 704 of the of the Amalgamated Association of Street, Electric Railway, and Motor Coach Employees of America went on strike in Little Rock, North Little Rock, and Cammack Village.

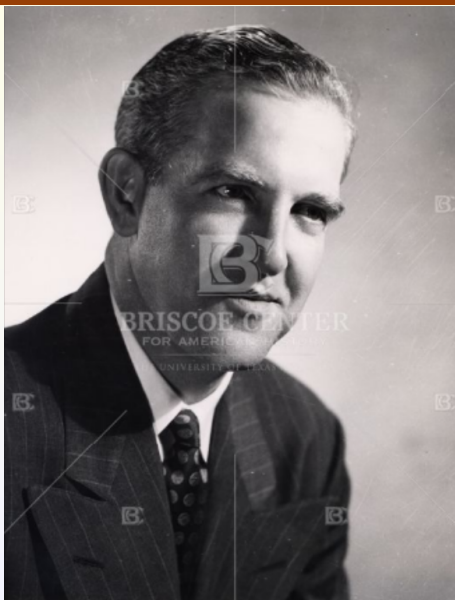
## Askey family travels, trials, and tribulations

- The employer Capitol Transit Company, and Division 704 had been unable to agree on an arbitrator to settle a wage dispute. ... When the company decided to hire “scabs” to operate the busses, violence erupted. ... A third attack occurred the same night, when a bullet struck the home of Phillip [sic] E. Askey, who had become president of CTC in October, after Norman Hill resigned and moved to Houston, Texas, to operate a truck line.
- According to Askey's uncle Phil, while Askey was an undergraduate, Askey did live in Arkansas during the summers that the Askey family lived there. Also, during one of those summers, Askey worked at a manufacturing plant, but didn't enjoy it.

## Askey–Washington University in Saint Louis–grader

- Askey also mentioned that Washington University had quite a reasonable library, and he used to haunt that starting from his freshman year.
- Askey told the math department that he knew some calculus and they gave him an oral exam. He answered all the questions, and they put him in the class the questions had been from. Askey: *I didn't understand at the time, but it was a very good thing, because the person teaching the class was Walter Leighton who was chair of the department.* Leighton immediately hired Askey as a paper grader and for the first three years he was grading three courses per semester.
- Askey commented: *Classes met three to five times a week, with maybe twenty students per class, ten homework problems a night per student, so I graded an awful lot of homework problems. [chuckles]*
- Askey: *The math department was a department that was confident and was willing to do things that were very much out of the ordinary.*

# Walter Leighton, Jr. (University of Missouri, Columbia)



## Askey–Washington University in Saint Louis–later years

- Askey took electricity and magnetism, physics, atomic physics, astronomy and skipped advanced calculus, and instead took complex variables, real analysis, modern algebra (out of Birkhoff–MacLane), and topology. Askey also sat in on a course on the theory of equations that Frank Haimo gave when he was a freshman. Askey said: *That was the last time they taught theory of equations. Can't say I learned very much, you don't learn much just by sitting in on something, you learn mathematics by actually doing.*
- Askey didn't care if he got bad grades. His mother had told him that grades weren't important, what was important was what you learned. He took this to heart. He emphasized that if he knew the material, he didn't care what kind of grade he got.



# Deborah Tepper Haimo (1921-2007) and Franklin (Frank) Haimo (1919-1982) both members of Princeton IAS



## Askey–Washington University in Saint Louis

- Askey tells a story where a professor who taught honors calculus said in regard to Askey, *I never had a student that deserved an A more and he'd never been happier to give the student a B. What's important is not what your potential is but what you actually do. I clearly knew all the material, but I was sloppy. He wasn't going to give me an A, he was going to give me a B to teach me a lesson. And it's a valuable lesson and I used that as an illustration to students when they would complain about their grades.*
- Askey: *I jumped beyond where I should have been, but I didn't know how to study mathematics, because I'd never had to study mathematics before in my life. It had always just come to me. Learning mathematics can be hard. And I was still learning how to learn mathematics.*

## Askey Transcript at Washington University in Saint Louis

**WASHINGTON UNIVERSITY**  
 ST. LOUIS, MO.  
 BALTIMORE CITY COLLEGE, MARYLAND, ENTERED 9-2  
 ADMITTED FROM

**WASHINGTON UNIVERSITY**  
 DECATUR, GA., 6-1-55  
 ADMITTED BY HIGHEST FOURTH WASHINGTON U. DECATUR, GA., 6-1-55

ENTRANCE UNITS: 2 1 2 1 1 1 1 1

DESCRIPTIVE TITLE	SECT	COURSE NO.	GRD	CRS	UNITS
MATH PROFICIENCY EXAM		113, 111	8		
LIBERAL ARTS FALL 51	156	53			
COMPOSITION&RECTORC	E CM	101	3		
CALCULUS I I	MATH	213	4		
ELEM. MECH. H&PROP	PHYS	211	5		
ELEMENTARY FRENCH I	FRNC	101	3		
BASIC COURSE I	MS&T	101	1		
LIBERAL ARTS SPR 52	156	53			
WAVE MOTION SOUND	PHYS	212	5		
BASIC COURSE I	MS&T	100	1		
CALCULUS III	MATH	214	4		
FIRST YEAR GERMAN I	GERM	101	3		
COMPOSITION&RECTORC	E CM	102	3		
LIBERAL ARTS FALL 52	156	53			
CHIEF ENG WRITRS	ELIT	211	3		
PRINCIPLES OF ECON I	ECON	211	3		
CONCEPT MATH ANALYS	MATH	417	3		
EDUCATIONAL PSYCH	PSY	209	3		
FIRST YEAR GERMAN	GERM	102	3		
MODERN ALGEBRA I	MATH	423	3		
BASIC COURSE II	MS&T	201	1		
		10	10		
LIB ARTS SPR 53	156	53			
CHIEF ENG WRITRS II	ELIT	212	3		
PRINCIPLES OF ECON2	ECON	212	3		
2ND YEAR GERMAN I	GERM	211	3		
HIGHER ALGEBRA II	MATH	332	3		
ADVANCED CALCULUS	MATH	413	3		
GENERAL PSYCHOLOGY	PSY	206	3		
BASIC COURSE II	MS&T	202	1		
		10	10		
LIB ARTS FALL 53	156	53			
PRIN OF LOGIC	PHIL	211	3		
ELECTRICITY	PHYS	423	3		
GREEK PHILOSOPHERS	PHIL	313	3		
ADVANCED CALCULUS B	MATH	414	3		
FUNCTNS COMPLEX VAR	MATH	421	3		
2ND YEAR GERMAN II	GERM	212	3		
		10	10		

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**WASHINGTON UNIVERSITY**  
 ST. LOUIS, MISSOURI

ASKEY, RICHARD ALLEN 15653  
 BIRTH DATE 6-1-33 BIRTH PLACE ST. LOUIS, MISSOURI

HOME ADDRESS ROUTE 3, BOX 260C, LITTLE ROCK, ARKANSAS  
 NAME OF PARENT OR GUARDIAN MR. PHILIP E. ASKEY  
 ADDRESS 8  
 BIRTH DATE 6-1-33 BIRTH PLACE ST. LOUIS, MISSOURI

DESCRIPTIVE TITLE	SECT	COURSE NO.	GRD	CRS	UNITS
LIB ARTS SPR 54	156	53			
STATISTICS I	ECON	231	3	B	6
FUND CON MATH AN II	MATH	418	3	A	9
THEORY OF FUNCTIONS	MATH	422	3	B	6
RESEARCH	MATH	393	3	A	9
GREEK PHILOSOPHERS	PHIL	314	3	C	3
ELECTRICITY II	PHYS	422	3	B	6
		10	10		39
LIB ARTS FALL 54	156	53			
GEN ASTRONOMY I	ASTR	201	3	A	9
TOPOLOGY	MATH	461	3	A	9
RESEARCH	MATH	593	3	A	9
INDEPENDENT WORK	PHIL	500	3	A	9
ELE ATOMIC NUCLEAR	PHYS	380	3	A	9
1170 1170 2300	015	15	15		45
LIB ARTS SPR 55	156	53			
GEN ASTRONOMY II	ASTR	202	3	A	9
MODERN ALGEBRA III	MATH	425	3	A	9
TOPOLOGY	MATH	462	3	A	9
RESEARCH	MATH	593	3	A	9
INDEPENDENT WORK	PHIL	500	3	A	9
1320 1320 2650	015	15	15		45

FINAL HONORS

1853

## Askey–Washington University in Saint Louis

- Askey dropped back and took advanced calculus during his junior year. Isidore (Izzy) I. Hirschman was teaching that course, and Askey was the paper grader. Hirschman must have recognized Askey's talent. Hirschman had gotten his Ph.D. at Harvard. Askey started in the middle of his junior year reading some things with Hirschman.
- Apparently, the first thing he read was David Widder's book on *Laplace transforms*. He says he didn't do enough problems, so he didn't really get it down cold. And when **Hirschman** and **Widder** were writing a book, Hirschman gave Askey the manuscript of the book, *The Convolution Transform*, which he read. David Widder, who was in the mathematics department at Harvard University was *sort of an old-fashioned mathematician*.
- Askey graduated from Washington University in Saint Louis with an Bachelor of Arts degree in mathematics on June 8, 1955.

# Isidore (Izzy) Isaac Hirschman Jr. (1922-1990) and David Widder (1898-1990)–Harvard University



## Askey–Washington University to Harvard University

- When Askey was a senior, Hirschman gave Askey a math problem to work on. He had given Askey a paper on a Legendre polynomial expansion theorem, and Hirschman thought that it could be extended to ultraspherical polynomials, which Askey did. They wrote a joint paper on this which was written when Askey was a senior which was ultimately published in *Transactions of the AMS*. Askey said that this paper had a big influence on his choice of research area, which at the time wouldn't have been clear to him.
- Essentially all of Askey's professors at Washington University had Harvard Ph.D.s: **Walter Leighton** who had been hired as chair a few years before Askey came in—they got along—(Leighton graduated from Harvard in 1935 under Marston Morse); **Thomas Luther Downs, Jr.**, who had given Askey his entrance calculus exam (his bridge partner, **Russell Andrew**); **George Booth Van Schaack** (who worked part time at the Missouri Botanical Gardens and did a taxonomic study of grasses); and **Franklin Haimo**.

## Askey–Graduate school–Harvard–1956

- During Askey's senior year, he applied to five schools: Harvard, Princeton, Chicago, Yale, and MIT. Askey was accepted (and enrolled) in the graduate program in the Department of Mathematics at Harvard University. Askey would have worked with **Dave Widder** (1898-1990) at Harvard, who was a student of **Birkhoff** and **Hirschman**'s advisor and collaborator and co-author, but Widder was on leave in Italy that year.
- According to Askey, the math department was 'cold'. It was too small for the size of the graduate program that they had — everybody (all the professors) kept their door shut. The professors would come out for a weekly colloquium tea but that was it. You had to make appointments to go see them.

## Askey–Harvard–the library issue–1956

- Askey used the library at Washington University quite extensively. To him, that was one of the main functions of what he got out of undergraduate school.
- He was finally at a place that had a good library where he couldn't wait to read everything that was in there. However, according to Askey, the library wasn't very user-friendly.
- The Harvard library had a duplicate collection in the math building, but you could not take books out from it. The duplicate collection was only open from about 9 to 5 when they had somebody working in the library. Askey had accepted an NSF Fellowship. Because of this, he apparently wasn't allowed to take journals out of the library (if he was a teaching Fellow then he could have).



## Askey–Master’s–Harvard–library issue–1956

- Askey on the Harvard library issue:

*I'd written a paper when I was an undergraduate and wanted to continue working on that area, and Xerox machines didn't exist at the time. I used to sleep late in the mornings. A group of us asked the department if they would leave the math library open in the evenings so they could use it. They said no, they'd have to have a librarian there. And I said no, we'll staff it. People can't take things out. They said no, people will steal books. And we said no, we'll station somebody at the door. And they said the townies will come in the building. We said, no, we'll put somebody at the door. And they said no, we'd have to heat the building. I said, we don't care about heat, we'll wear our coats. We just want access to your books and journals in the evening. They said no, we'd have to leave the lights on. I said to myself, okay, I'm going elsewhere.*

- Askey was at Harvard for one year and obtained a Master's of Arts degree from Harvard University in 1956.

## AT&T Bell Labs in Murray Hill, New Jersey (Summers of 1956, 1957)

- The summer between Harvard University and Princeton University, Askey worked at Bell Labs in Murray Hill, New Jersey. He also worked there the following year. The first year Askey served as an applied mathematician and worked with electromagnetic waves. He says didn't do anything of importance there.

Askey: *Either summer, really. I learned some things.*

## Askey–Ph.D. study at Princeton University (1956-1961)

- Askey recontacted Princeton: *I made a mistake. I should have gone here last year when you accepted me. I wonder if you would consider me again?* They did and Askey transferred to Princeton.
- Askey: *Princeton was a completely different place. The quality at Princeton was significantly higher; the faculty was much larger. They left people alone. There were some very good people at Harvard, but not nearly the high fraction that there was at Princeton. At Harvard, I knew more mathematics than fifty percent of the graduate students I knew, in every area of mathematics. At Princeton, there wasn't a single graduate student that I dominated in every area of mathematics.*
- Askey: *There were people like Barry Mazur [1937-] who started graduate school when I went to Princeton, he was eighteen at the time.*
- At Princeton, graduate students had keys to the library building, and the library itself didn't have a lock on the door. Askey: *You had a key to the building so you could use the library at any time you wanted.*

## Askey–Ph.D. study at Princeton University–1956-1961

- *Askey: Princeton is totally different than Harvard. Harvard was exam-oriented, written exams in two languages. And some were oral exams. You didn't take an exam for a master's, but you had to take a prelim exam. Written at Harvard, oral at Princeton. At Princeton, the basic philosophy was that you're responsible for learning the basic, the undergraduate courses. Most undergraduate students have had a lot of this already. And graduate students that don't know it, you might offer a course in one of these things, but you might not. If you don't, then you get together with two or three other graduate students and you learn it on your own. So the students were given much more responsibility. And this was a mixed blessing. Some people thrive in that, and some people don't.*

## Askey at Princeton University (1956)–Arne Beurling

- Askey made contact with Princeton professor **Arne Beurling**, a Swedish analyst, in regard to finding a research problem.  
*Askey: Beurling gave me a thesis problem which I couldn't solve and I never have solved it, and it still hasn't been solved, it hasn't ever been mentioned in print. Beurling had solved the problem on the integers and he wanted to have it solved on the real line. I was afraid to ask him how to do it on the integers because it might restrict my way of thinking about the problem on the real line.*
- Askey befriended **Mark Kac**, Polish-American mathematician, Cornell for six years. Kac influenced Askey's career. Mark Kac had told *Askey: You could never tell how good a Beurling student was until that student had been out a couple of years because all Beurling students wrote superb theses, but you don't know how much of the thesis that the student did and how much Beurling did. Askey said: I wanted to find out what I could do on my own, so I never really got much help from Beurling, and I never solved his problem [laughs].*

# Arne Buerling (1905-1986) and Mark Kac (1914-1984)



Howard S. Cohl (NIST)



Richard Allen Askey

## Askey at Princeton University (1957-58)–Salomon Bochner

- In the second year at Princeton, Askey came up with an idea for a thesis topic and went back to Princeton that summer and talked to Galizien [Austrian-Poland] born Princeton analyst and differential geometer **Salomon Bochner**. Askey: *He would have been the natural person for me to work with if I hadn't gotten tied up with Beurling.*
- Askey: *One of the reasons that I decided to go to Princeton was the people they had doing probability theory such as William Feller. I was afraid that if I just did analysis without some motivation from somewhere else I would end up being sterile. So I wanted to see if I could use probability theory as a place to get intuition. Then I got to Princeton, and Salomon Bochner said that it took him ten years of studying probability theory before he got enough intuition before it helped him in his analysis, and then I heard these marvelous lectures of Beurling that didn't have any motivation from anywhere else at all. It was just pure analysis, so I said the heck to that.*

# Salomon Bochner (1899-1982)





## Askey at Princeton University (1957-58)–Salomon Bochner

- In regard to Bochner and Askey's chosen research project,  
Askey: *...and every time I would see him, I would have to start from the very beginning. Because it wasn't that close to his main area of interest. He had some interest in it and he knew some things about it.*
- Askey: *The summer before passing my oral, I'd been back in Princeton telling Bochner what I was doing, and he did something that was perfect for me. He didn't give me a problem to work on, but what he did do was give me two problems to work on afterwards, that were significantly harder than what I'd done before. And you can't ask more from a thesis advisor than that, good problems to work on after you get out. So, this can help deepen you.*

## Askey at Princeton University (1957-58)–Salomon Bochner

- *Years later...* [I think this was at the “Problems in Analysis: A Symposium in Honor of Salomon Bochner” conference held April 1-3, 1969] Askey: ... *when I gave some lectures, in the introduction I thanked Bochner for being an ideal thesis supervisor. He left me alone to write a thesis. You need something that’s going to push you farther. And they were both very nice problems, and they both got solved.*
- Askey was at Princeton for two years and then he got married to Liz Askey. He felt that he needed a job to support his wife. Askey: *It’s a good thing, because my first child was born in the year we got married. So I took a job as Instructor at Washington University for three years before I finished my degree and came up with a problem on my own.*
- Note that Askey’s first child, Jim Zurlo was born on February 24, 1959 in Saint Louis, Missouri. Askey’s second child Suzanne Askey was born on November 16, 1961, in Chicago, Illinois.

## Princeton–Washington University Instructor (1958-1961)

- *Askey: Hirschman had actually come up with a problem for me first. Then he was actually involved in the solution too much. So that wasn't really feasible as part of a thesis. I did a little bit of work on it and then he came up with a solution to the whole thing. I'd done some of the work, too. So that was also a joint paper. But it wasn't part of the thesis.*
- *Askey: So, I was back at Washington University and I was sort of floundering there. And finally did come up with an idea and I did write up a thesis. Askey was an instructor at Washington University for three years.*
- *Askey's thesis work never ended up being published in research papers.*

## Askey's Princeton Ph.D. Thesis (January 1961)

## MEAN CONVERGENCE OF ORTHOGONAL

## SERIES AND CONJUGATE SERIES

BY

RICHARD ALLEN ASKEY

A DISSERTATION  
PRESENTED TO THE  
FACULTY OF PRINCETON UNIVERSITY  
IN CANDIDACY FOR THE DEGREE  
OF DOCTOR OF PHILOSOPHY

RECOMMENDED FOR ACCEPTANCE BY THE  
DEPARTMENT OF  
MATHEMATICS

JANUARY, 1961

## ABSTRACT

Let  $w(x) = (1-x^2)^\lambda$ ,  $\lambda > -1/2$ , and  $p_n(x)$  be the sequence of polynomials orthonormal with respect to  $w(x)$  dx on  $[-1, 1]$ . Denote by  $L_w^p$  the space of measurable functions on  $[-1, 1]$  such that

$$\|f\|_{p, w} = \left( \int_{-1}^1 |f(x)|^p (1-x^2)^{-\lambda} w(x) dx \right)^{1/p}$$

is finite. For  $f \in L_w^1$  we write

$$f'(x) = \int_{-1}^1 f(t) p_n'(x) w(t) dt$$

and

$$s_n(f; x) = \sum_{k=0}^n f'(x) p_k(x).$$

Pollard [*Trans. Amer. Math. Soc.*, vol. 63 (1948), pp. 353-367]

has shown that

$$\lim_{n \rightarrow \infty} \|s_n(f; x) - f(x)\|_{p, w} = 0$$

for all  $f \in L_w^p$  if  $\lambda < (\lambda+1)/(2\lambda+3) < p < \lambda/(\lambda+1)/(2\lambda+1)$ . This result fails for some  $f \in L_w^p$  if the above conditions on  $p$  do not hold. We show that for  $1 < p \leq \lambda/(\lambda+1)/(2\lambda+3)$  this result fails because of the behavior of  $f$  at one and minus one. We have that

$$\lim_{n \rightarrow \infty} \|s_n(f; x) - f(x)\|_{p, w} = 0$$

for all  $f \in L_w^p$  if  $1 < p \leq \lambda/(\lambda+1)/(2\lambda+3)$  and  $\omega < 3\lambda/4 - 1/p + \lambda/(1/2 - 1/p)$ . This result fails for some  $f \in L_w^p$  if  $\omega < 3\lambda/4 - 1/p + \lambda/(1/2 - 1/p)$ .

Define a conjugate function to  $f$  by

$$\tilde{f}(x) = [w(x)]^{-1} \int_{-1}^1 \frac{w(y) f(y)}{x-y} dy.$$

Then if  $q_n(x)$  denotes the usual second solution to the differential equation satisfied by  $p_n(x)$  we can prove the following theorem.

If  $f \in L_w^p$ ,  $1 < p < \infty$ ,  $\omega < 3\lambda/4 - 1/p + \lambda/(1/2 - 1/p)$ ,  $\omega \leq \delta$ ,  $1/\lambda - 1/p + \lambda/(1/2 - 1/p) < \delta < (\lambda+1)/p'$ ,  $p' = p/(p-1)$ ,  $\delta \leq \tau$ , and  $\lambda/p' - 1/p < \tau$  then

$$\lim_{n \rightarrow \infty} \left\| \sum_{k=0}^n f'(x) q_k(x) - \tilde{f}(x) \right\|_{p, w} = 0.$$

The above results all hold for a class of polynomials which includes the Jacobi polynomials. In addition we prove an  $L^1$  analogue to the first theorem.

Also we extend results of Wing [*Amer. J. Math.*, vol. 72 (1950) pp. 792-808] on mean convergence of Fourier-Bessel series in the same fashion as above.

## Princeton University—oral examination (1960)

- Askey: *I went back in the middle of my third year to Princeton to take the [oral] exam. I took the train up to Princeton. It was snowing. So we started the exam. And after about fifteen minutes, Bochner said, "You know, that snow's getting pretty bad. I think I'm going to leave. The rest of you carry on." Well, he did. I passed.*
- Askey: *But in retrospect, I started thinking about the final oral. The final oral is not usually an exam of the student. It's an exam of the professor. If you fail somebody on a final oral, you're in essence asking the professor to resign. Because you're saying that this did not meet scholarly standards within the field. And the professor better know that if he doesn't. You don't tell students that beforehand. Now, yes there can be a subtle error in a thesis. It's caught there. So it's not 100%. But it was Bochner walking out of my exam before it was over and saying we're done, as far as I'm concerned, that later made me think about this.*

## Askey–University of Chicago (1961-1963)

- *Askey: I then went to Chicago for a couple of years as an instructor, met Steve Wainger, a colleague here at Wisconsin now, Steve was telling me what it was he was working on and that he was stuck at a certain point. And where he was stuck was exactly something that I knew how to do. So we started working together, and the two of us together solved the two problems of Bochner. I said in the foreword to my regional conference lectures that I don't know whether Steve could have solved them without me; I know I couldn't have solved them without him, so that was a very fortuitous thing. I've been lucky all along. I ended up at good universities with good people to talk to all the time.*

## Askey–University of Chicago (1961-1963)

- Askey: *Yeah, but I'd been at Washington University for three years and went back to Princeton. Then I went to Chicago as an instructor for two years.*
- At the University of Chicago, Askey connected with famous analyst and harmonic analyst **Antoni Zygmund**. He was responsible for creating the *Chicago school of mathematical analysis* together with his doctoral student Alberto Calderón.
- Zygmund was author of the two-volume masterpiece in analysis *Trigonometric Series*. It was described by Robert A. Fefferman as “one of the most influential books in the history of mathematical analysis”

# Antoni Zygmund (1900-1992)–Polish Analyst (U. Chicago)

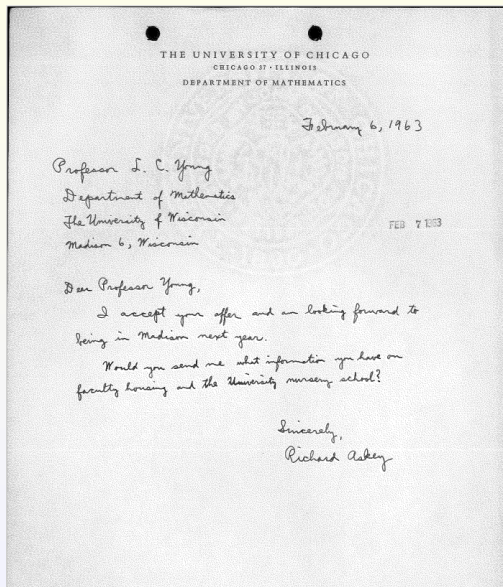




## Askey hired at UW-Madison (1963)

- Saunders Mac Lane (University of Chicago) indicated to Dick that Wisconsin was a good place for him: *I think Wisconsin would be very good for you. They treat people who are narrow and deep, very well.*
- Askey applied to Wisconsin-Madison and requested three letters of recommendation:  
Salomon Bochner, I. I. Hirschman, Jr., and Antoni Zygmund.  
(These letters have never been seen.)
- January 16, 1963. Askey was recommended for Research Assistant Professorship.
- January 23, 1963. Letter written to Askey offering him a position from L. C. Young
- February 27, 1963. Askey accepted the offer.

# Askey acceptance letter at UW-Madison



# Askey positions at the University of Wisconsin-Madison

- Held position of **Research Assistant Professor** (1963-1965)
- Held position of **Associate Professor** (1965-1968)
- Held position of **Professor** (1968-1986)
- Held position of **Gábor Szegő Professorship** (Houses) (1986-1995)
- Held position of **John Bascom Professorship** (1995)
- Became **Professor Emeritus** (2003-2019)

## Tiny fraction of the honors that Askey received

- **Guggenheim Fellowship** (1969-1970) (one year in Amsterdam)
- **Virginia Tech Conference Board of the Mathematical Sciences-NSF Regional Conference Series in Applied Mathematics** (1974) which resulted in the publication in 1975 of his famous monograph *Orthogonal Polynomials and Special Functions*.
- **College of Science Lectures**, Penn State University (1984)
- **Trjitzinsky Lectures**, University of Illinois (1984)
- **Vice President of the American Mathematical Society** (1986-87)
- **Honorary Fellow of the Indian Academy of Sciences** (1988)
- **Turán lectureship** in Budapest, Hungary (Spring 1992)
- **Elected Fellow of the American Academy of Arts and Sciences** (1993)
- **Elected to the National Academy of Sciences** (1999)
- **Fellow of the American Association for the Advancement of Science (AAAS)** (1996)

# Portrait of Askey (Elected vice President of the AMS)



## Dick Askey (1970) in The Netherlands



# Dick Askey (University of Wisconsin-Madison)



Howard S. Cohl (NIST)



Richard Allen Askey

# Bruce Berndt, Liz Askey, Alladi Ramakrishnan, George Andrews, Krishna Alladi and Dick Askey





Steve Milne, David Bressoud, Mourad Ismail and George Andrews. Second row: Dennis Stanton is behind Mourad



Doron Lubinsky, Paul Nevai, Dick Askey, Tom Koornwinder  
attending OPSFA-1 Polynômes Orthogonaux et  
Applications, Bar-le-Duc, France, October 1984.



# Dick Askey and Dennis Stanton



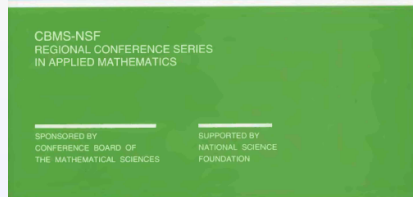
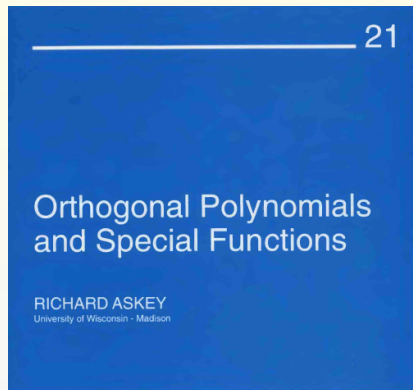
## Dick Askey's view on 'big problems' and 'problem solving'

- *Askey: I've never worried about "quote" big problems. The problems that I worked on have led to other problems. And there have been times in my life when I didn't know what I was going to do next. But in the course of my work, always something came up that would push me off in another direction.*
- *Askey: My basic advice to students has always been if you see how to do a problem, don't bother to think about how to do it, just do it. If you have trouble with it, then you start thinking. And if you turn out to end up with a surprising answer, after having done it, and it looks sort of like a miracle has happened, something unexpected has happened, and things are nicer than you would have expected, then you ought to start thinking: What is it that makes this nice? Because there may be some structural reasons for that and that's worth knowing. But even back then, if you didn't know how to do a problem, you do it!*

## Regional Research Conference in Blacksburg, Virginia 1974

- One of the first extremely important conferences (at least for me, Charles Dunkl) was the one in Blacksburg, Virginia from June 10-14, 1974. It was the “Regional Research Conference on Special Functions” at the Virginia Polytechnic Institute and State University (Virginia Tech).
- Ismail has indicated that this conference was really important in that it brought many people together and started some hugely important research efforts in Special Functions by people who hadn't been working in that area. Ismail continued by saying that Izzy Hirschman attended this meeting and at this point it was very clear that Askey had gone much further than Hirschman ever did.
- The principal speaker was Richard A. Askey. The participants of the meeting included several people who would become major contributors to the theory. These included Willard Miller, George Gasper, Mourad Ismail, and George Andrews. It was a fabulous event for young researchers (the younger people) to network.

# Askey's SIAM (1975) monograph



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## Beautiful Askey quote from his 1975 monograph

- This was pointed out to me and Sarah Post by Paul A. Martin...
- Askey: *Part of the secret of success in studying and using special functions is to try to remember exactly what is necessary, and nothing more.*
- **Richard Askey** (1933-2019), *Orthogonal Polynomials and Special Functions*, Society for Industrial and Applied Mathematics, Philadelphia, 1975, p. 9.

## Conclusion

# STOP

...and remember that sometimes mathematical meteors such as Dick Askey move through the atmosphere and pass us by. We just have to sit back and witness their passage and wonder and appreciate all the possibilities that are afforded to us by these rare events and deeply value their lives because they have impacted us so!