Talk plan

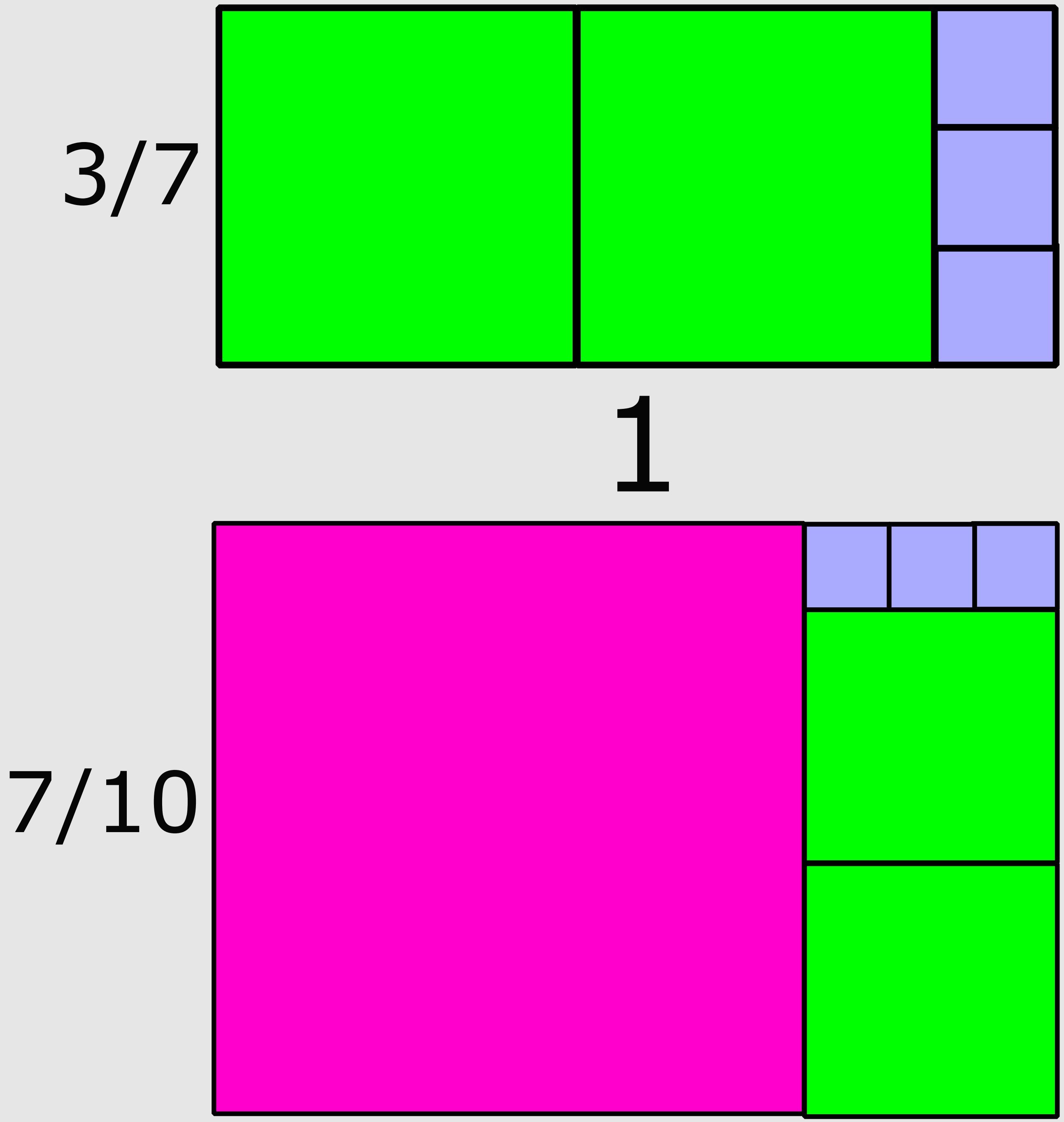
1. continued fractions
2. 4-color theorem
3. The Eisenstein integers
4. Triangulations w/degrees 6,...,6,2,2,2
5. Triangulations w max degree 6

Continued Fractions:

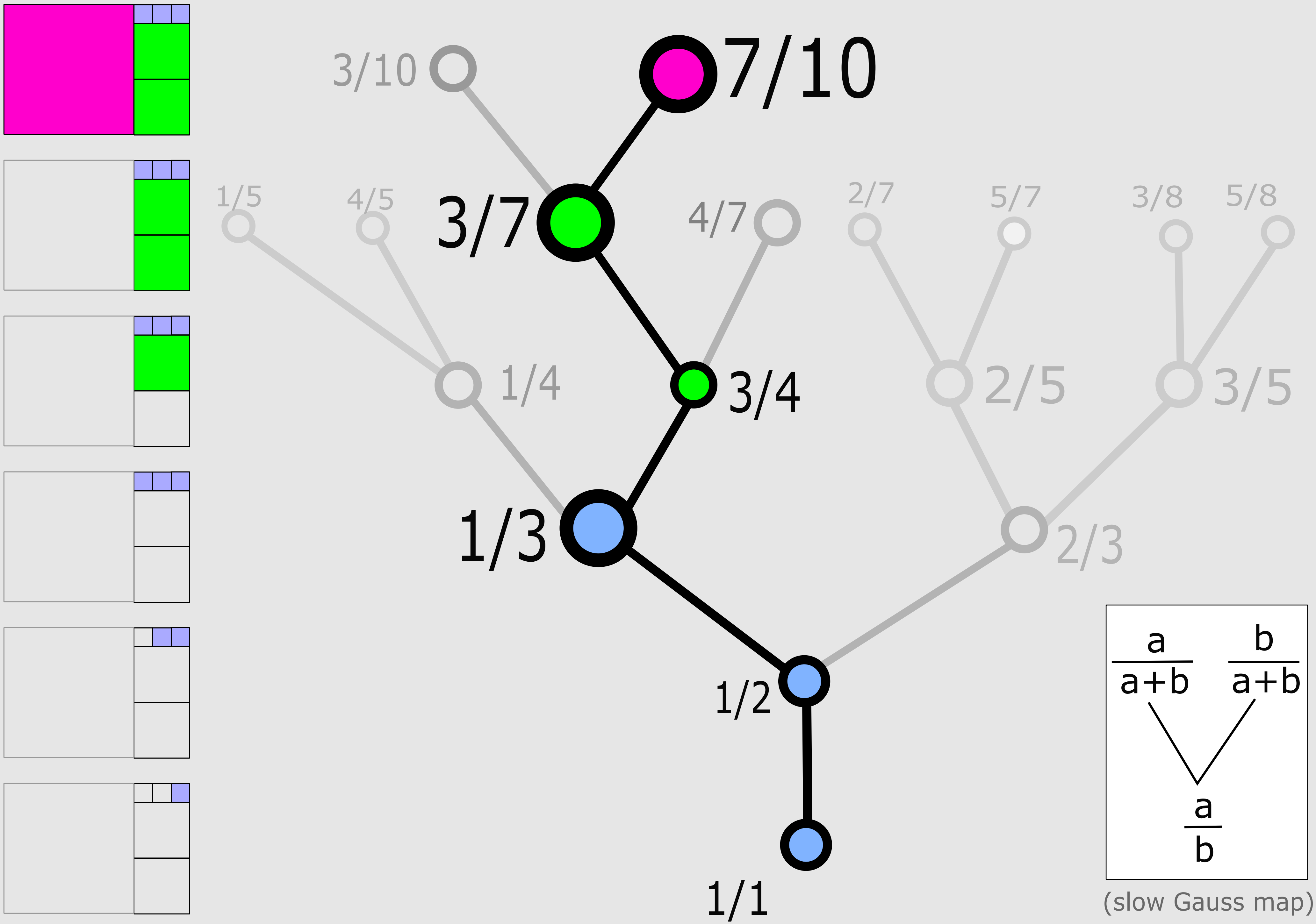
$$3/7 = \cfrac{1}{7/3} = \cfrac{1}{2+\cfrac{1}{3}}$$

$$7/10 = \cfrac{1}{10/7} = \cfrac{1}{1+3/7} = \cfrac{1}{1 + \cfrac{1}{2+\cfrac{1}{3}}}$$

geometric view of cont. fractions.



tree view of continued fractions



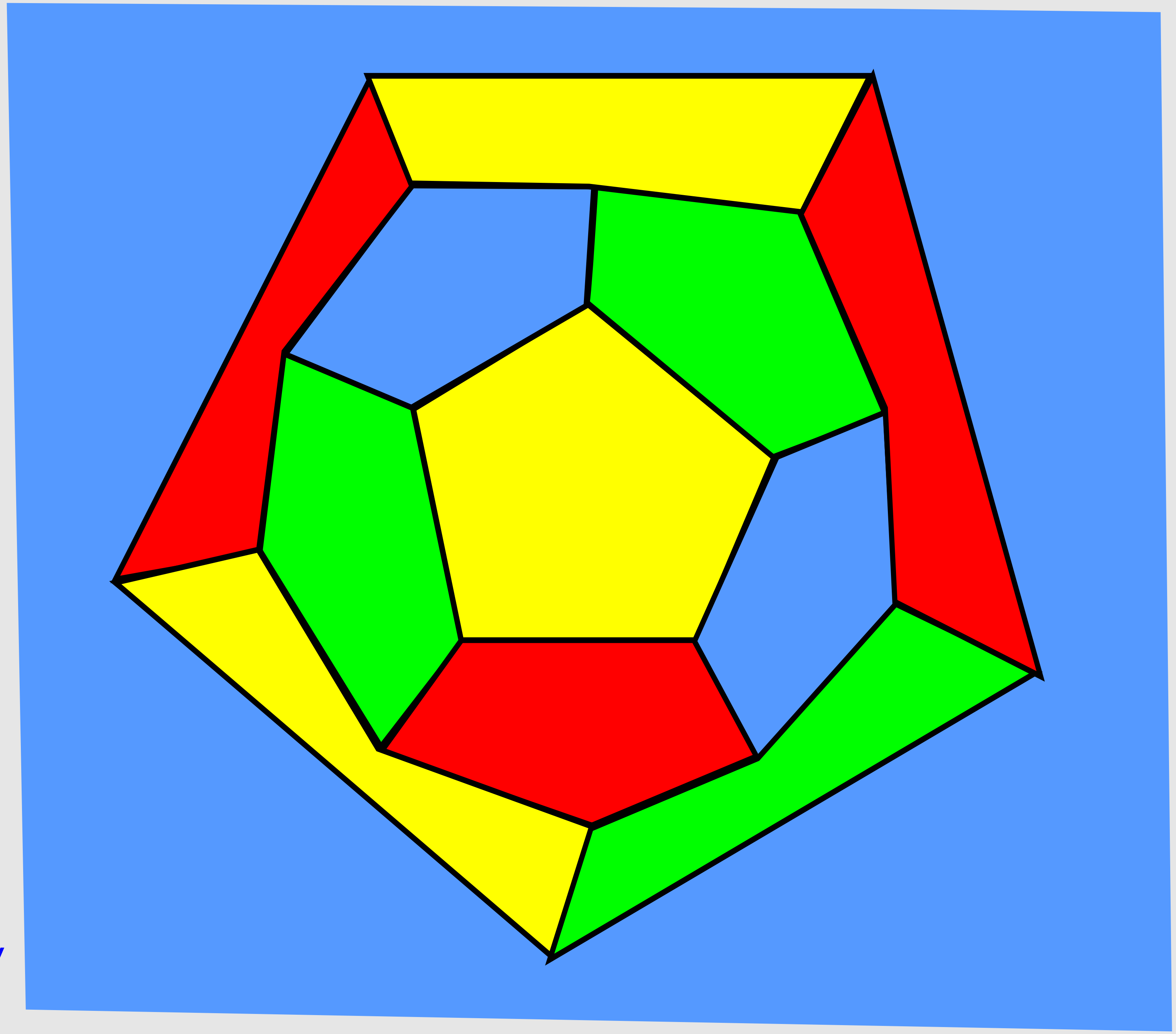
4-Color theorem (popular version)

The regions of
a planar graph
can be colored
with 4 colors so
that adjacent
regions get
different colors.

computer assisted
proof by
Haken and Appel
(late 70s)

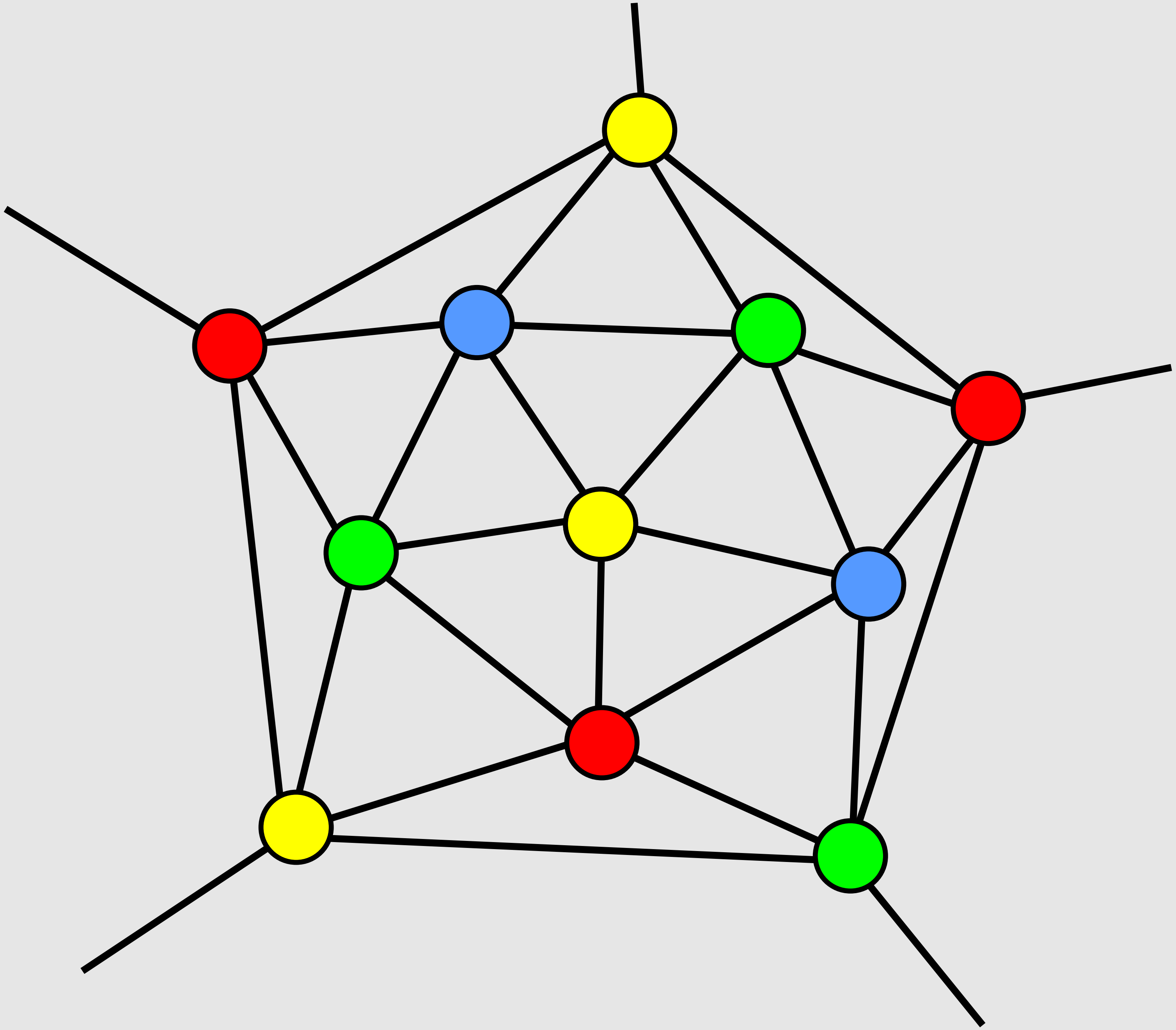
Improved by
Robertson and
Seymour (90s)

Proof checked (?) recently
by an automated checker.



4-Color theorem (dual version):

The vertices of a planar graph can be colored with 4 colors so that adjacent vertices get different colors. (It suffices to prove this for triangulations.)

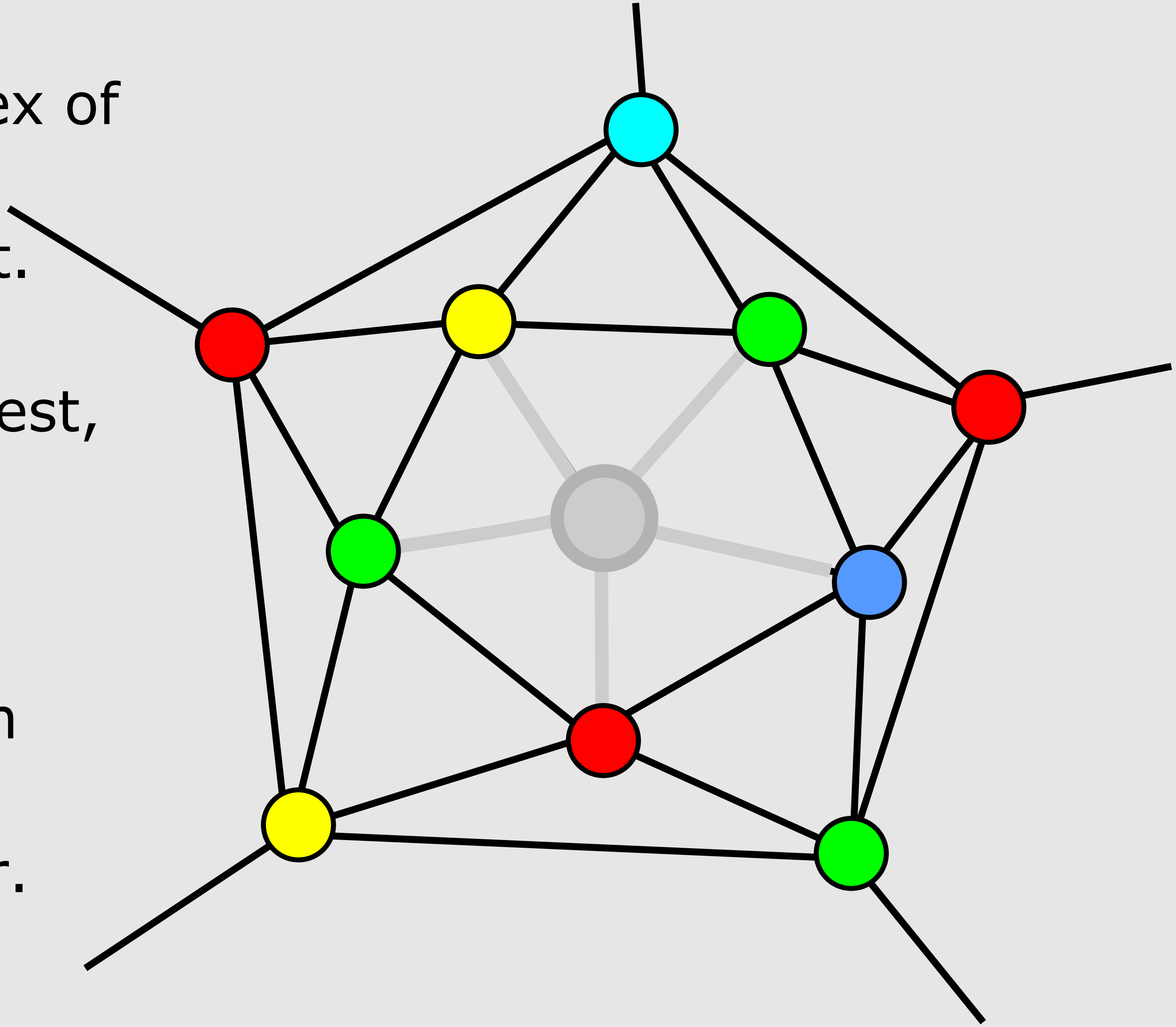


Proof of the 6 color theorem..

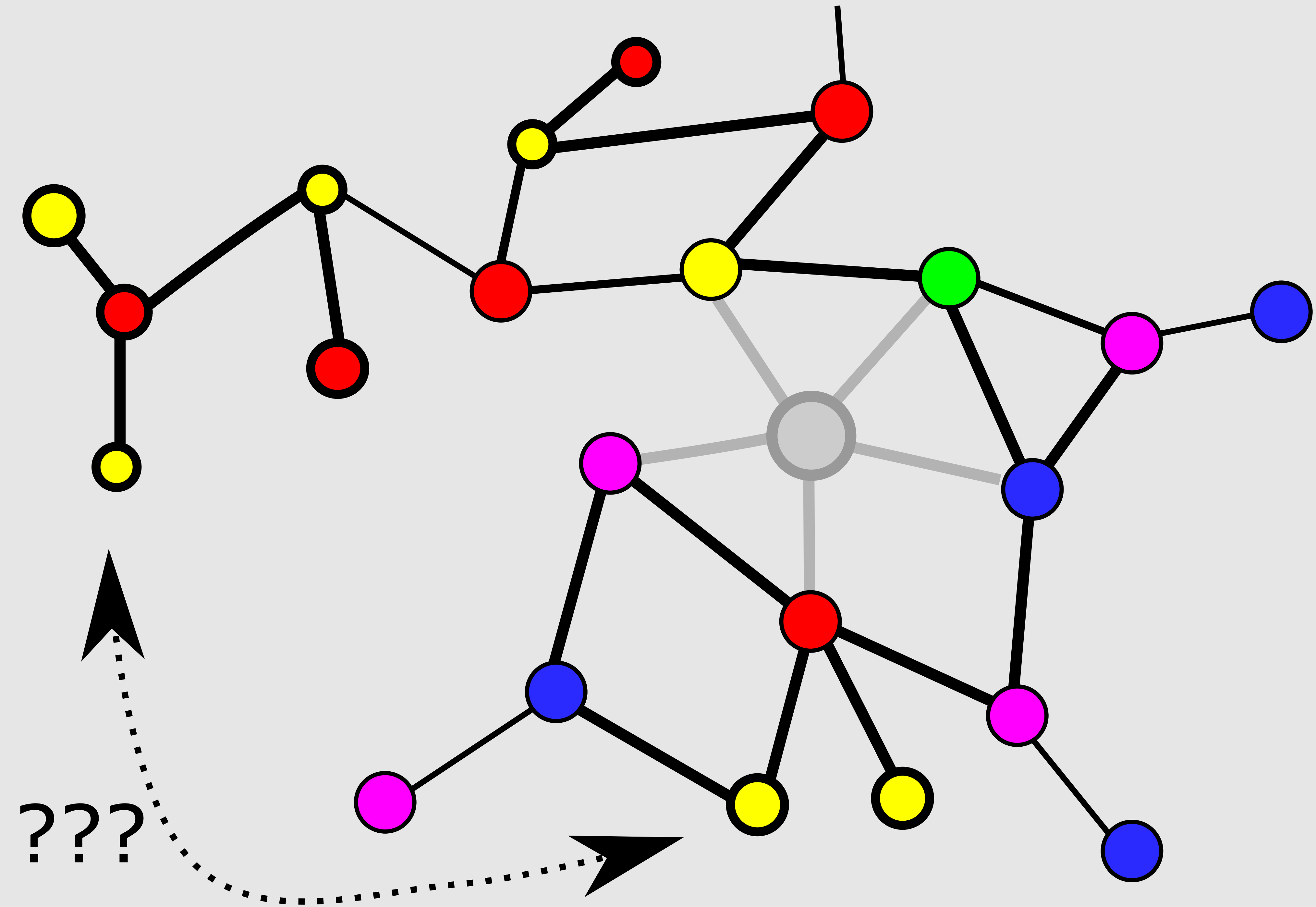
1.Find a vertex of degree <6 and remove it.

2. Color the rest, by induction.

3.Put the vertex back in and use the left over color.



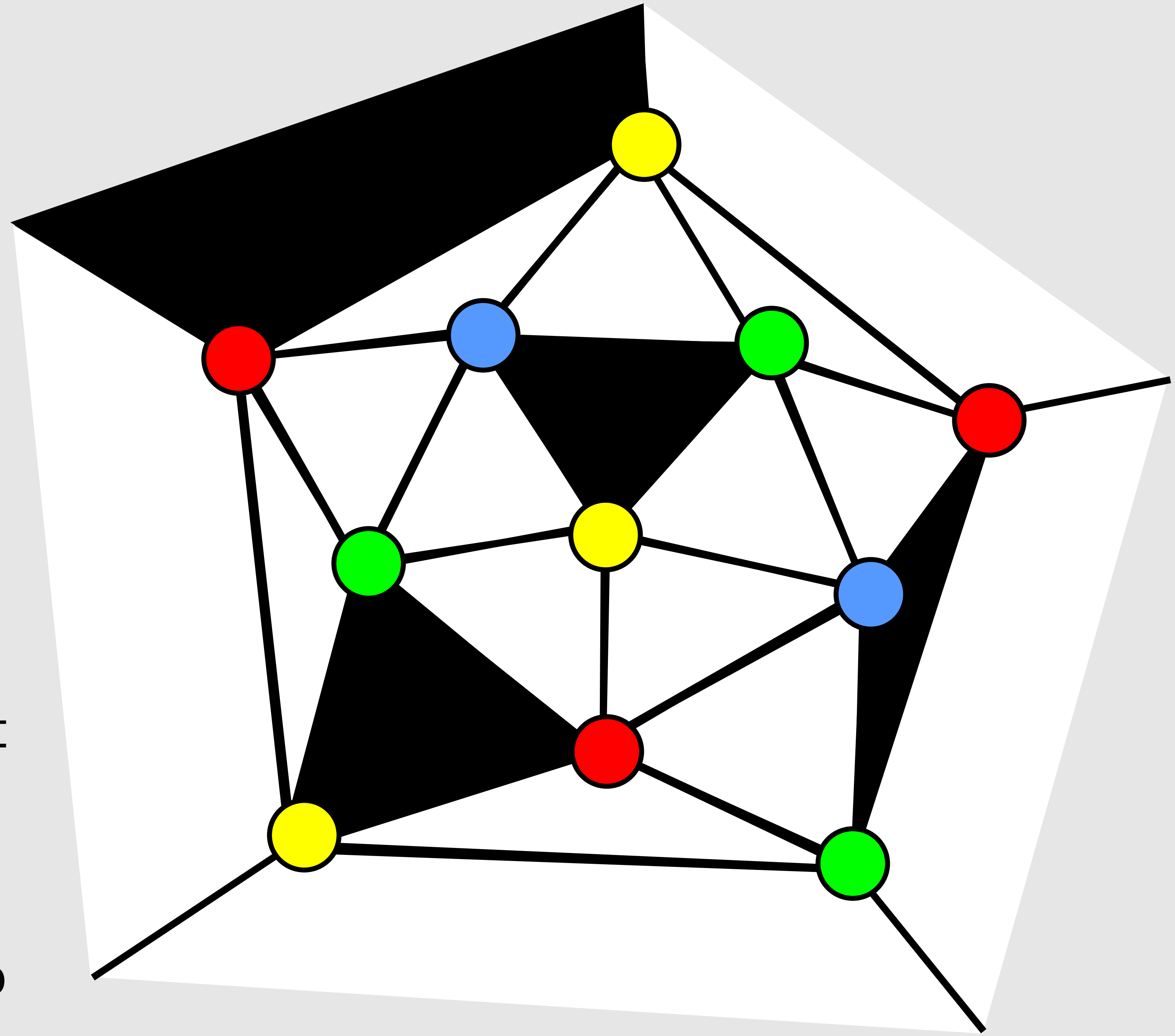
Proof of the 5 color theorem (Heawood):.



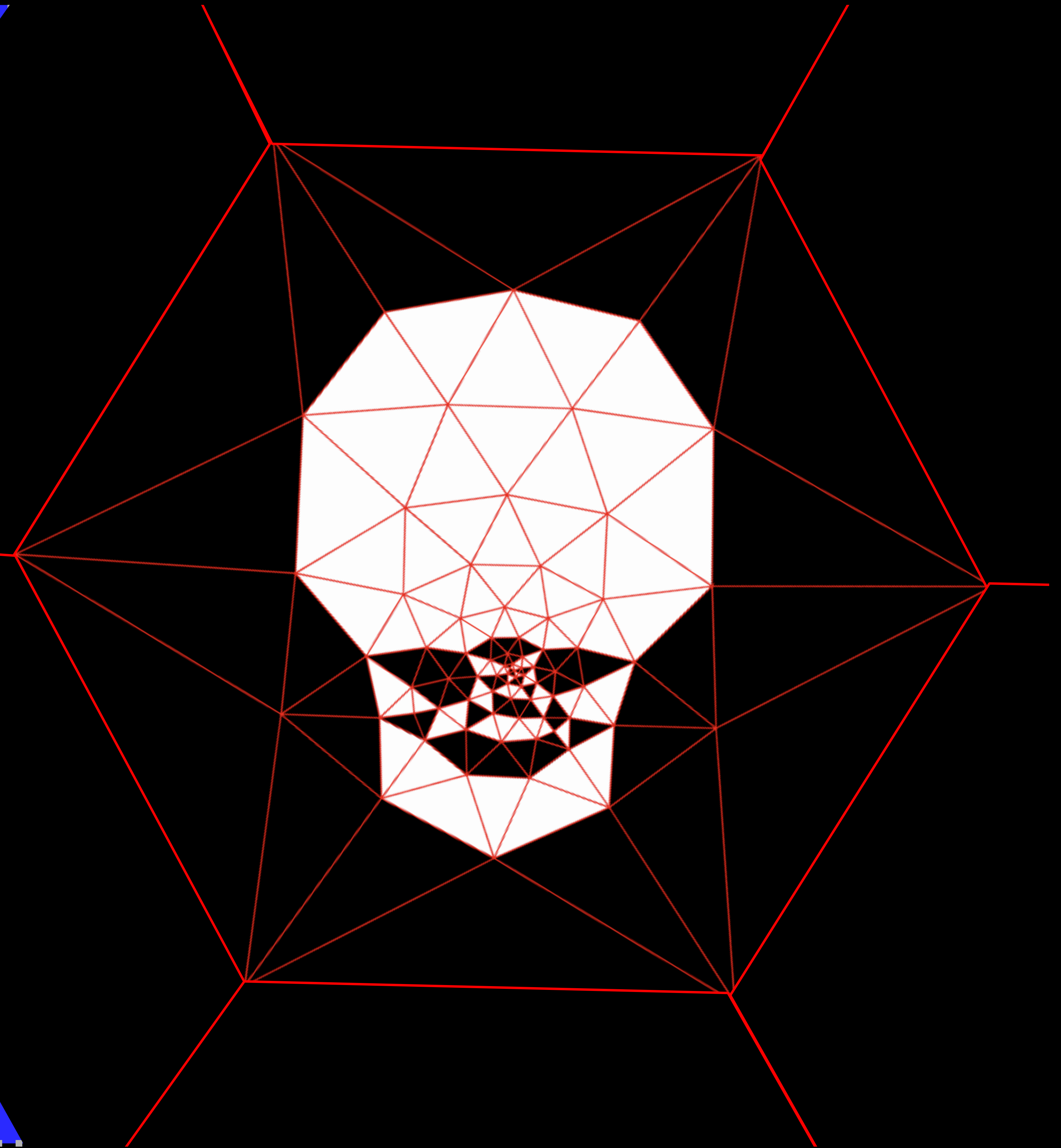
4-Color theorem (geometric version):

The faces of a triangulation can be colored black and white so that around each vertex #B and #W are congruent mod 3.

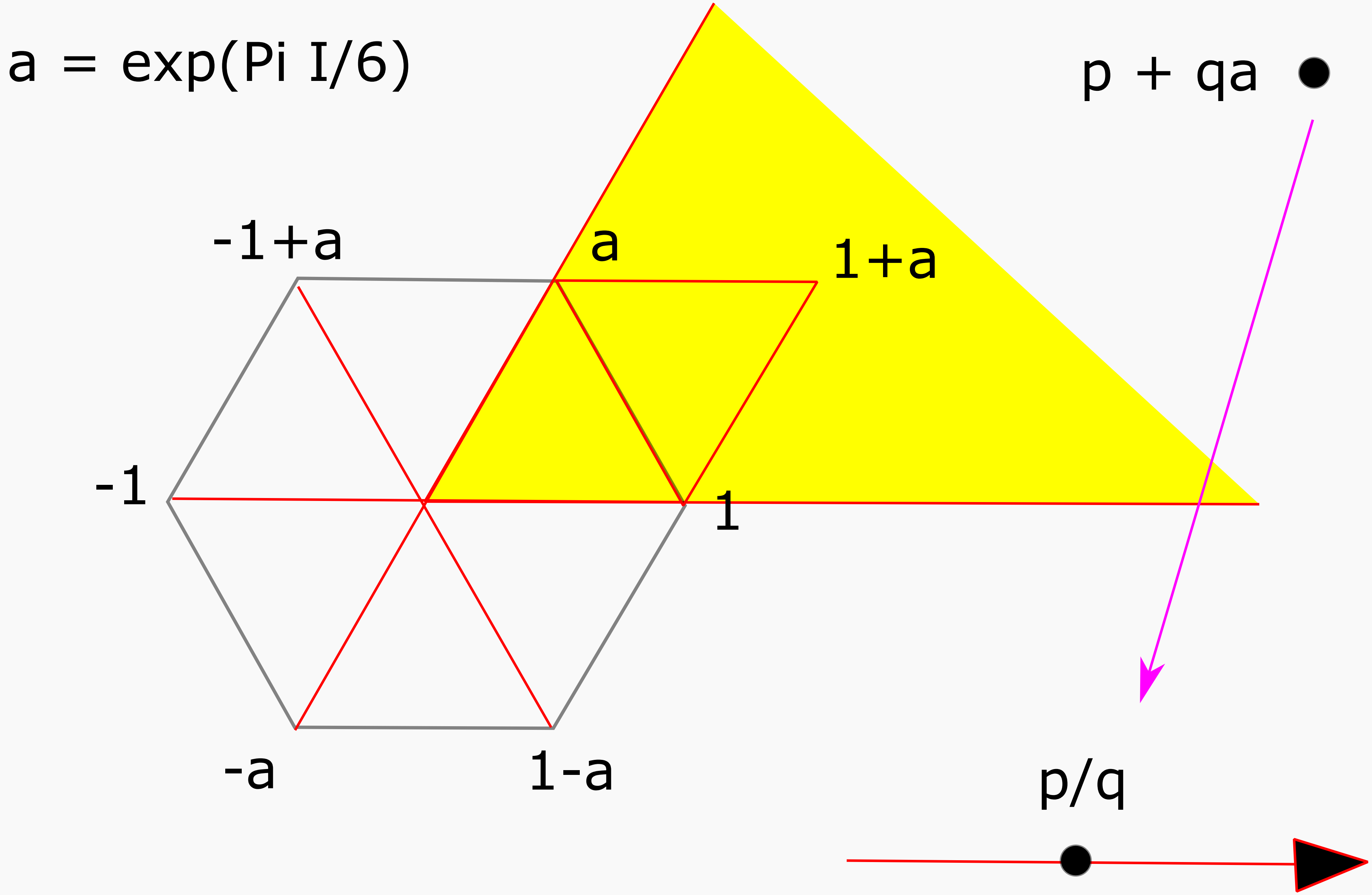
Geometrically, this is equivalent to the existence of a nowhere collapsing simplicial map to a tetrahedron.



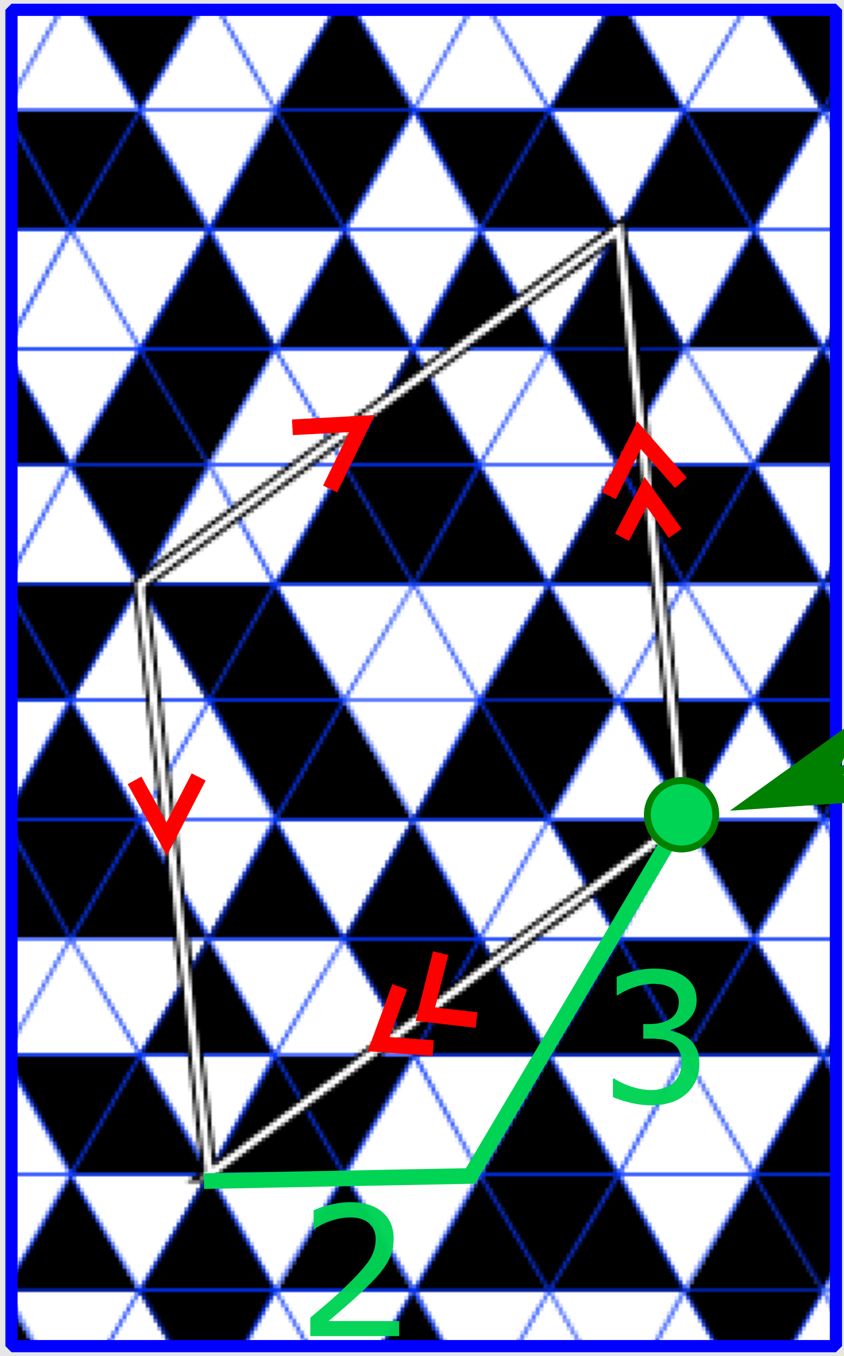
A halloween themed example



Eisenstein integers



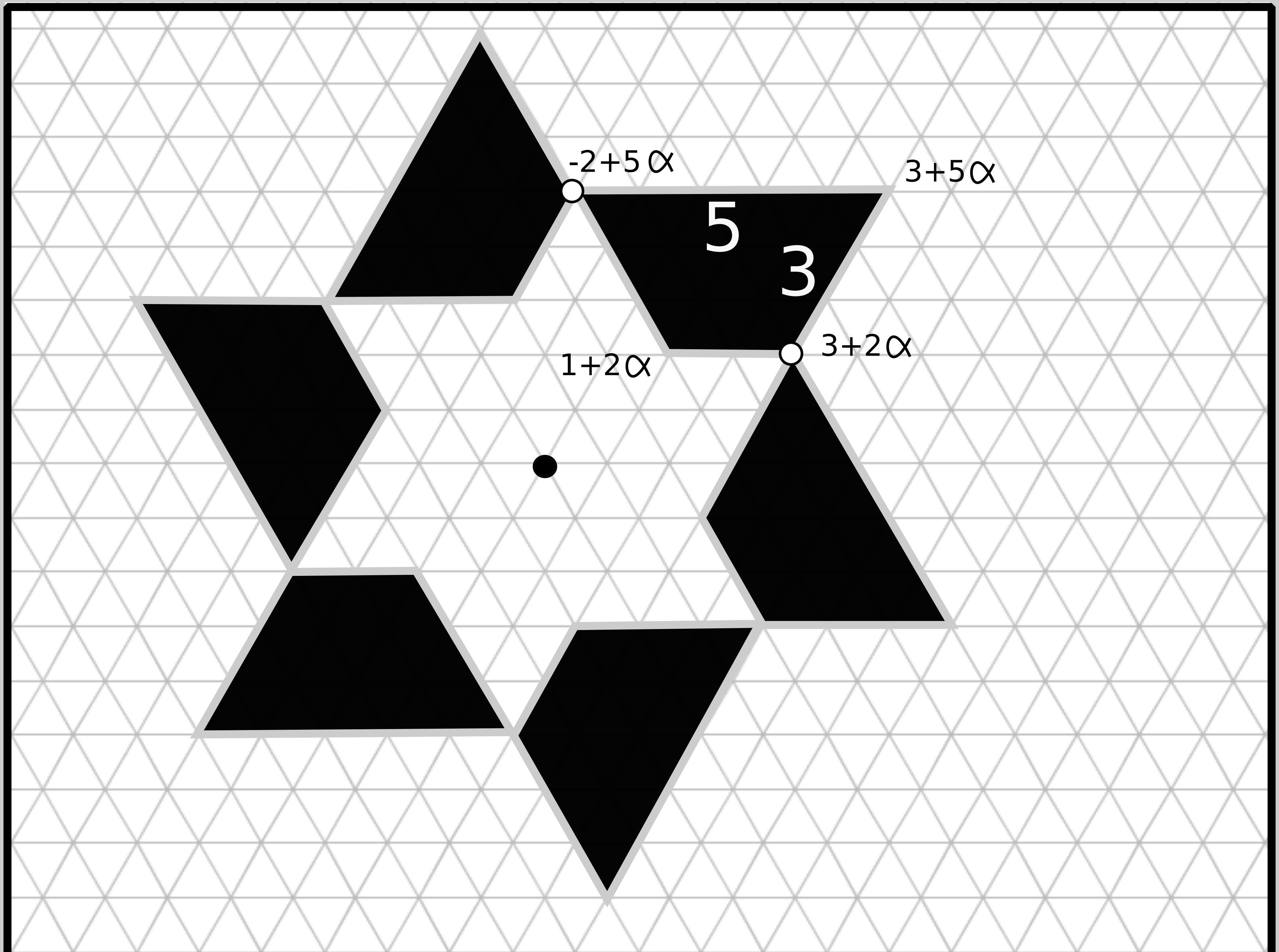
Triangs. with Deg sequence 6,...,6,2,2,2



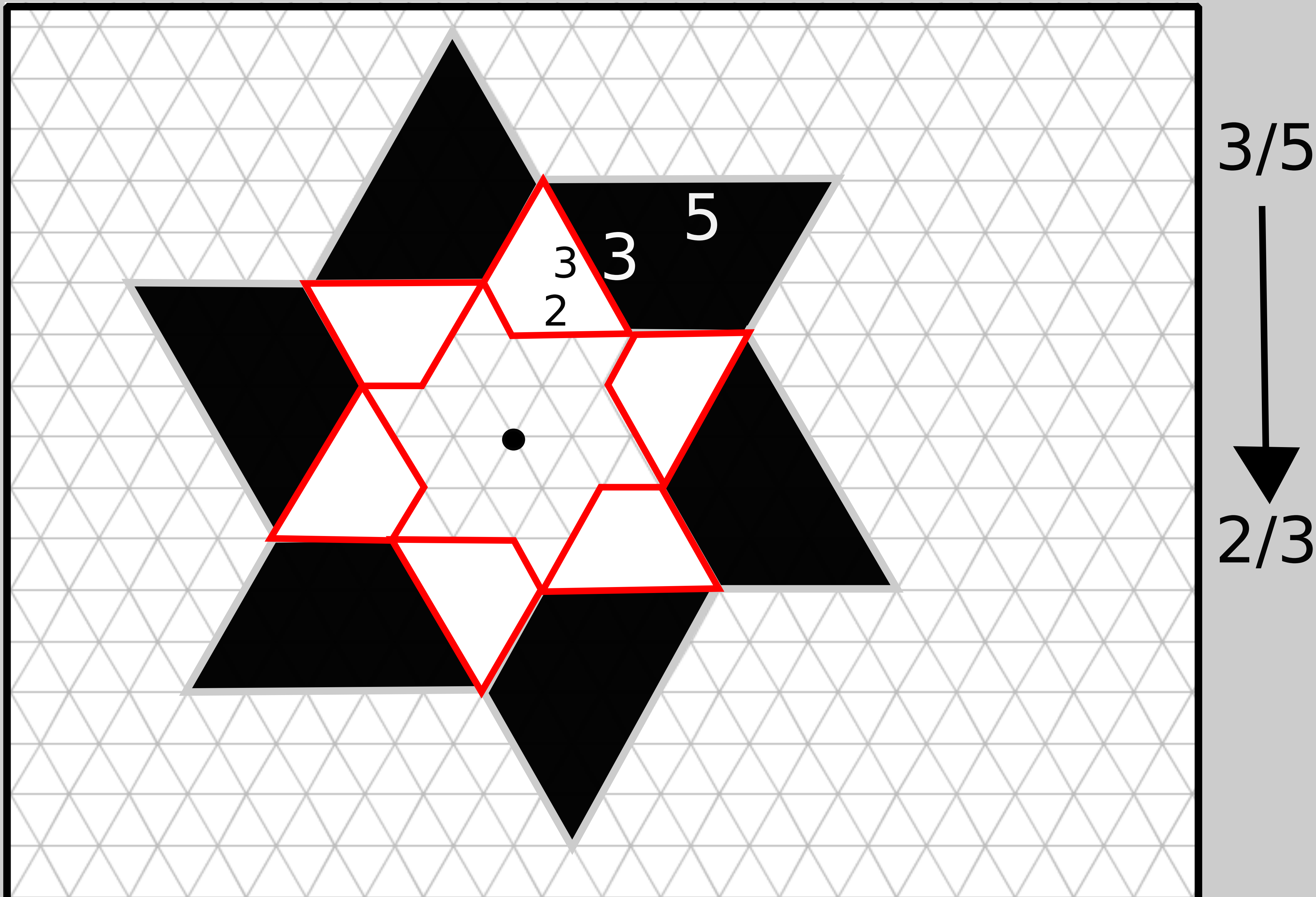
This is a coloring of T(2/3)

2 + 3 a

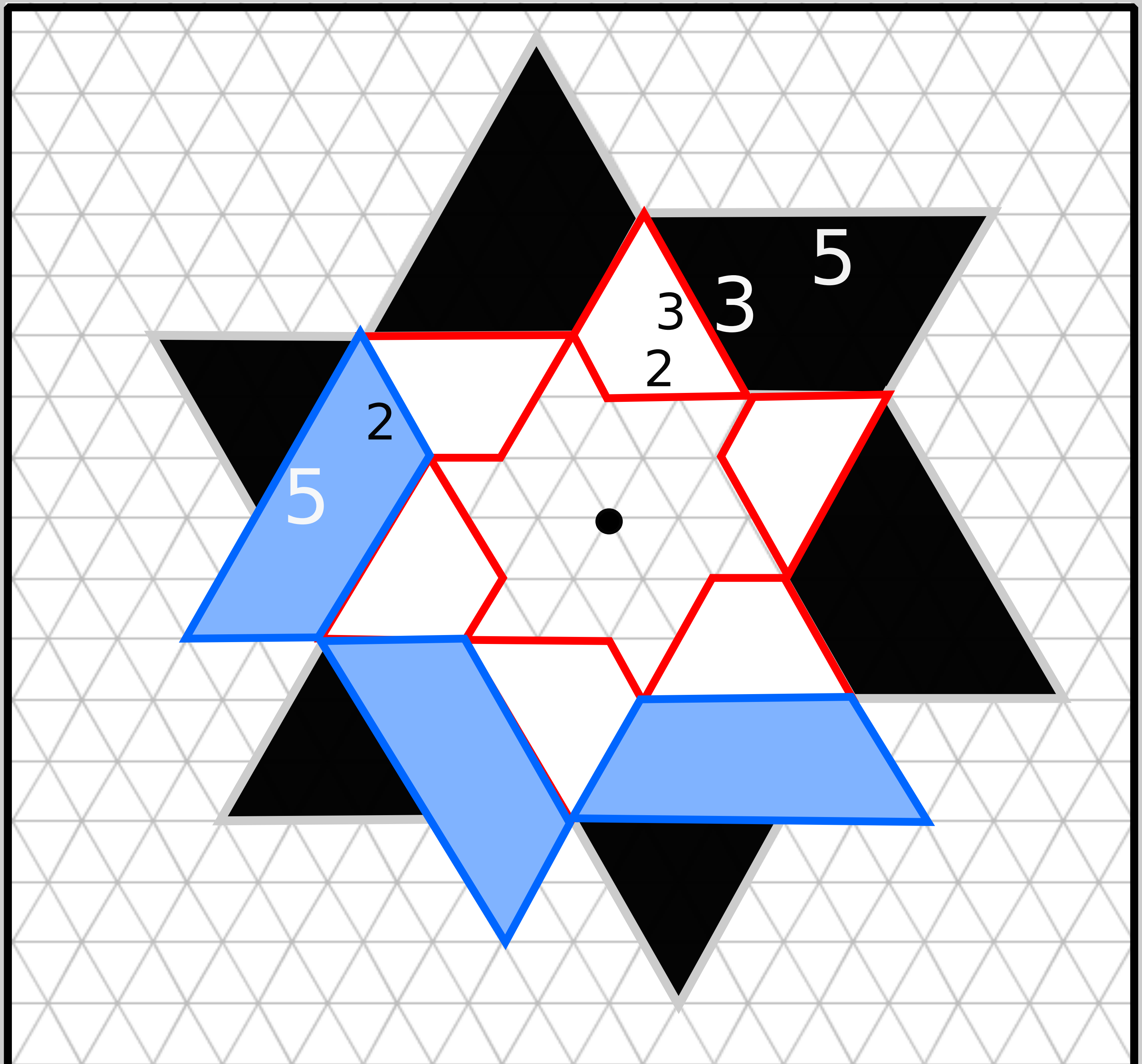
This is a trapezoid necklace

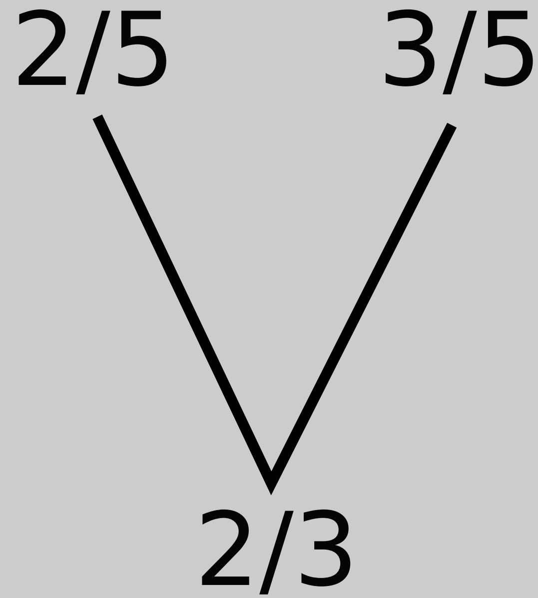


A big neckace entails a smaller one.



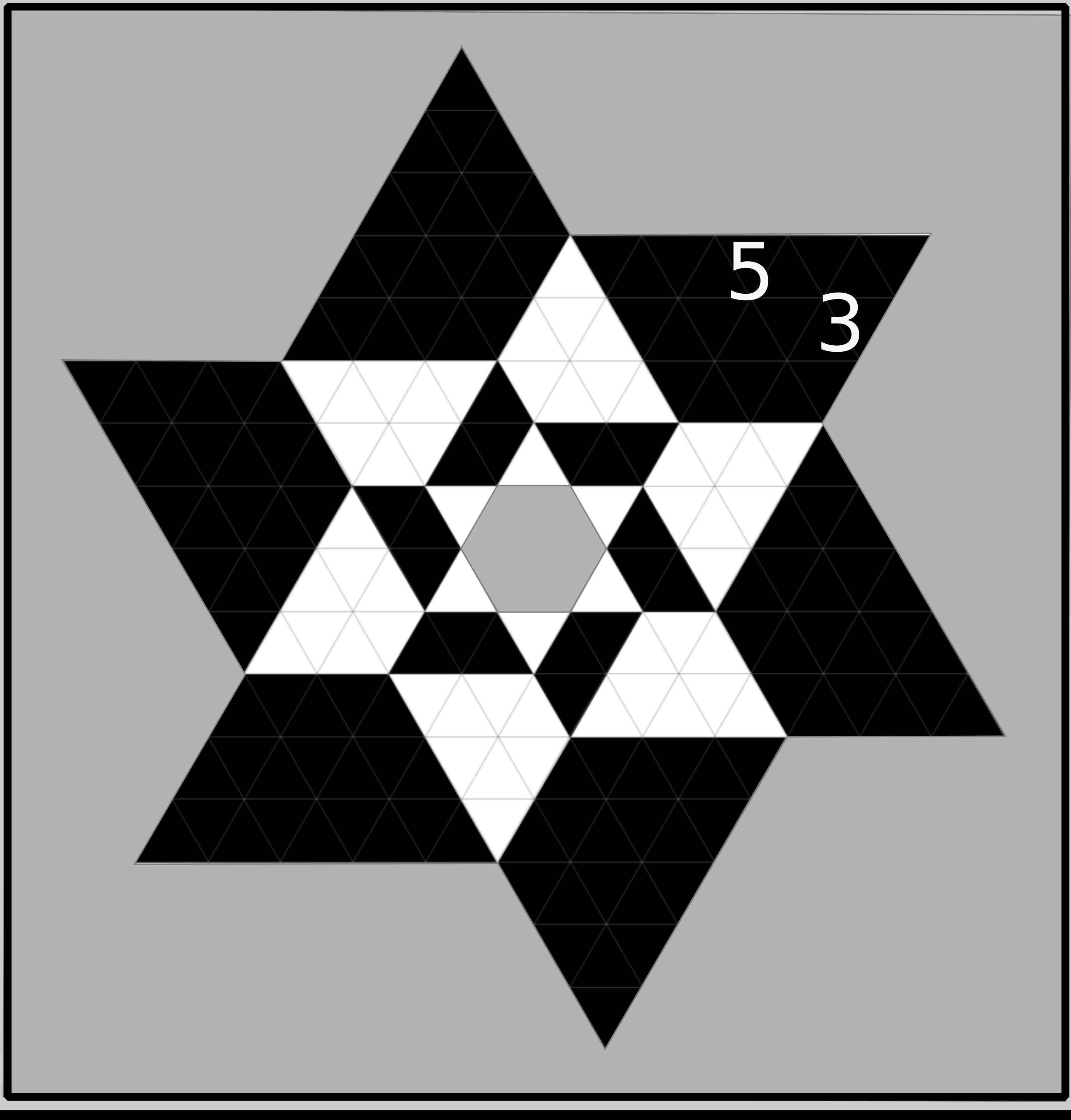
2 big ones entail the same small one.



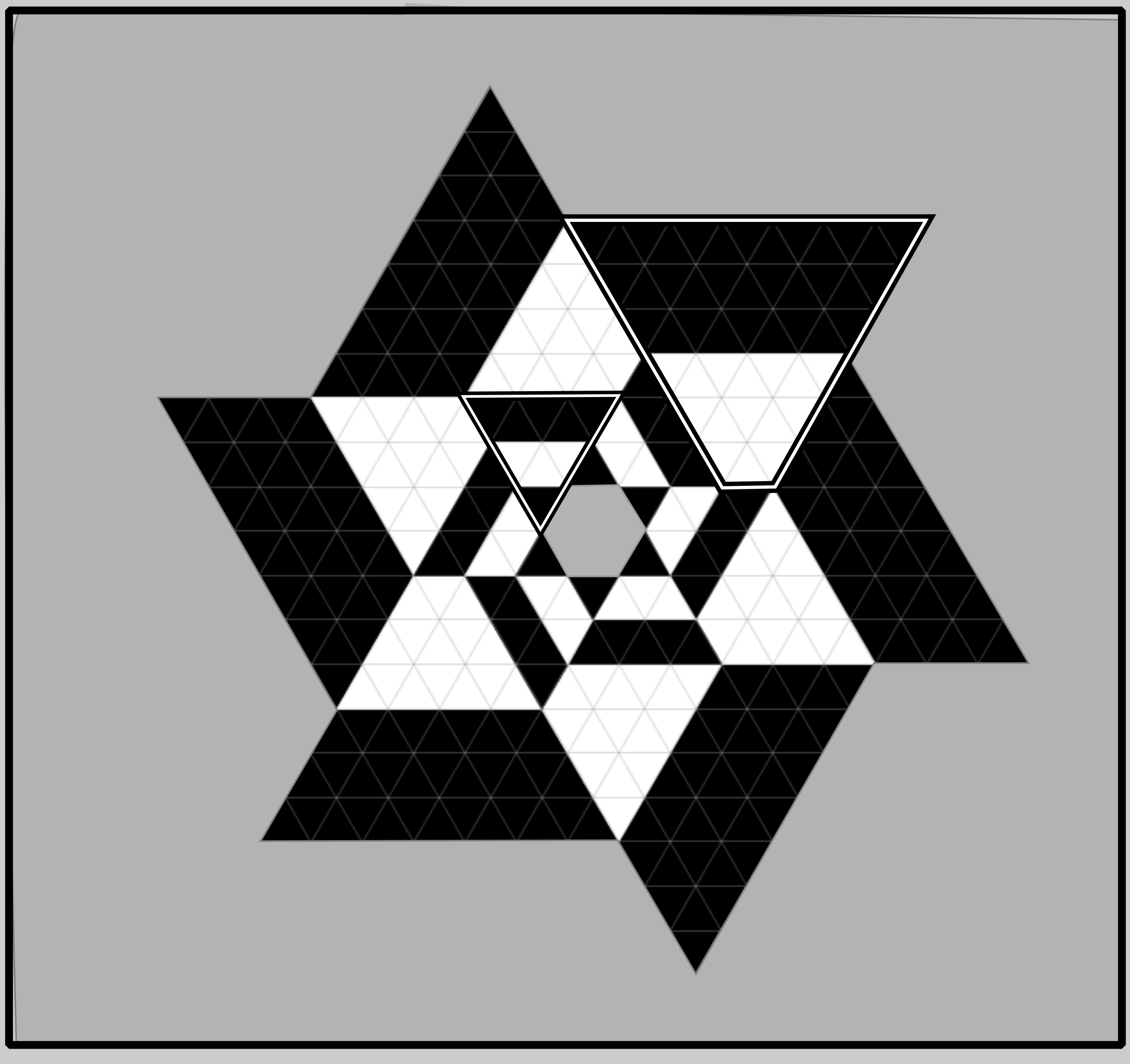


This operation does the slow Gauss map!

The empty 3/5-flower

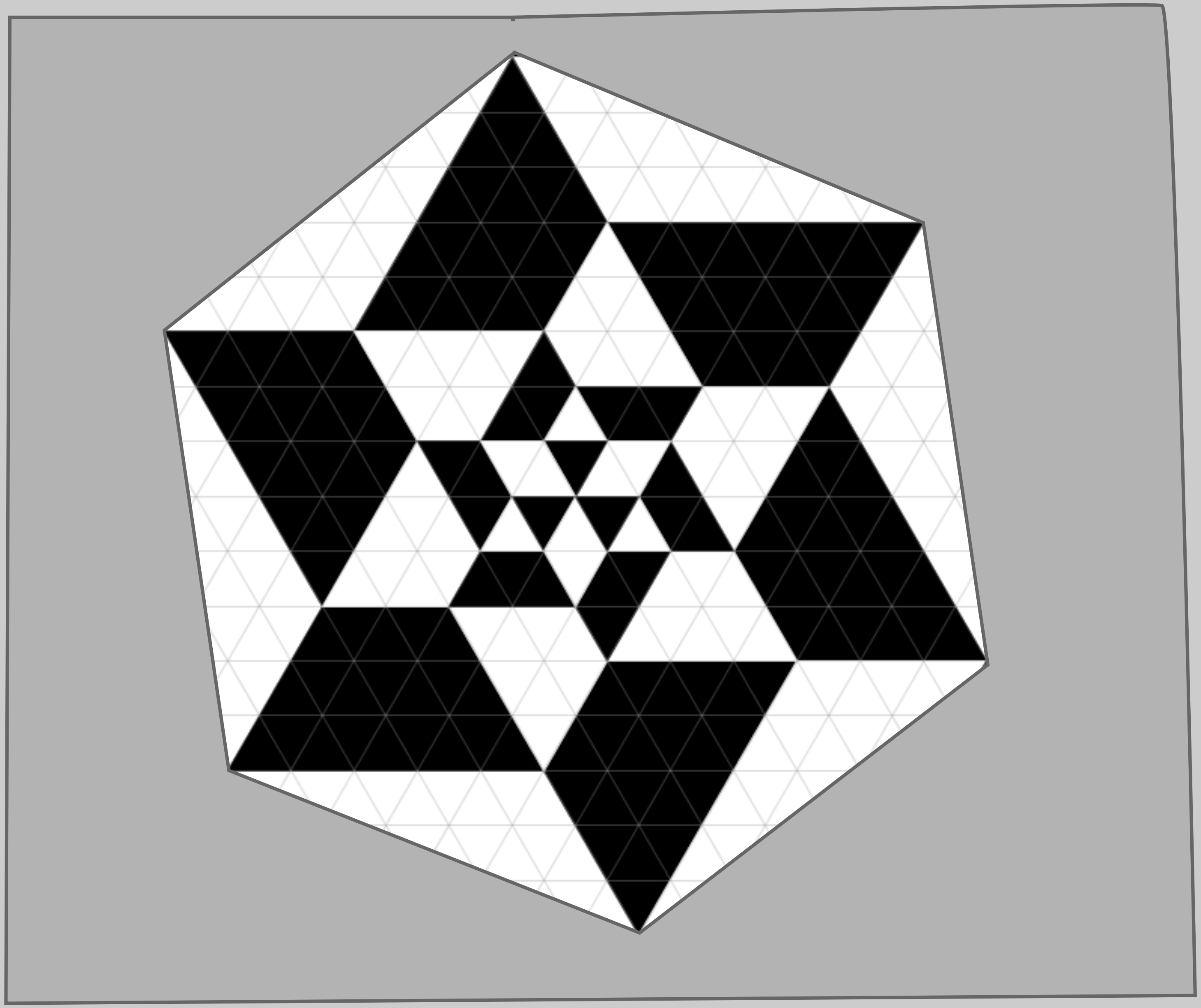


The empty 3/7-flower

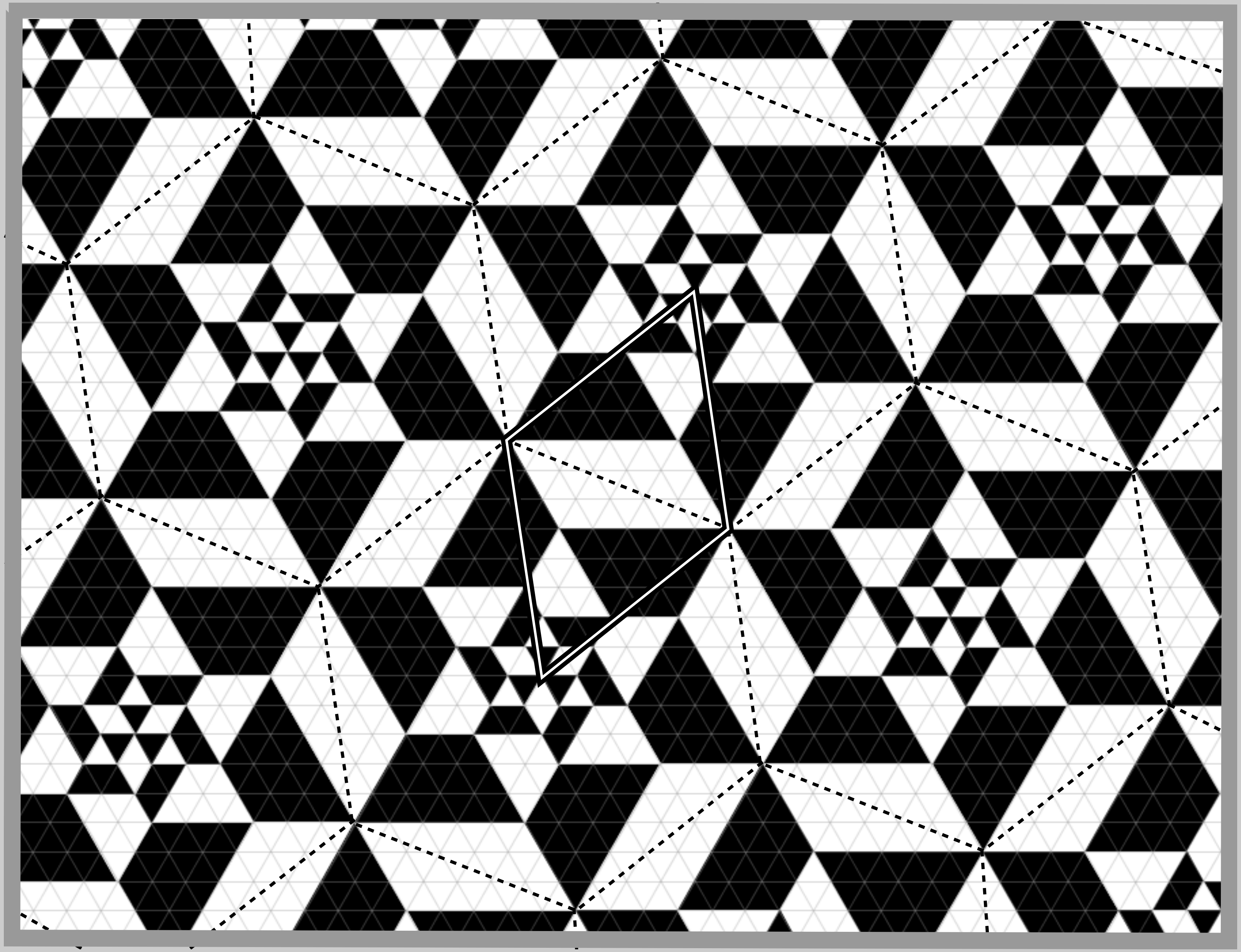


$$3/7 = \cfrac{1}{2 + \cfrac{1}{3}}$$

The 3/5-flower

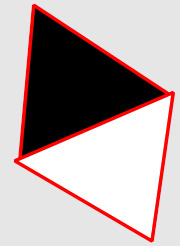


The 3/5-coloring



Properties:

Let f = number of B/W edges:



- if $\{p_n/q_n\}$ has an irrational limit then f_n/F converges to 0.

- if $\{p_n/q_n\}$ are the continued fraction approximants of a quadratic irrational, f_n^2/F has a well-defined limit η.

Def: η is the isoperimetric ratio of the quadratic irrational.

Sample calculations

$$\eta(\phi^{-1}) = \phi^{+6}, \qquad \phi = \frac{\sqrt{5}+1}{2}.$$

Let $\psi_n = \sqrt{n} - \text{floor}(\sqrt{n})$.

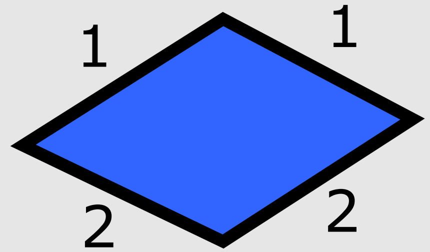
$$\eta(\psi_2) = \frac{75+53\sqrt{2}}{7} \qquad \eta(\psi_3) = \frac{132+72\sqrt{3}}{13}$$

$$\eta(\psi_5) = \frac{321+137\sqrt{5}}{19} \qquad \eta(\psi_6) = \frac{27+9\sqrt{6}}{2}$$

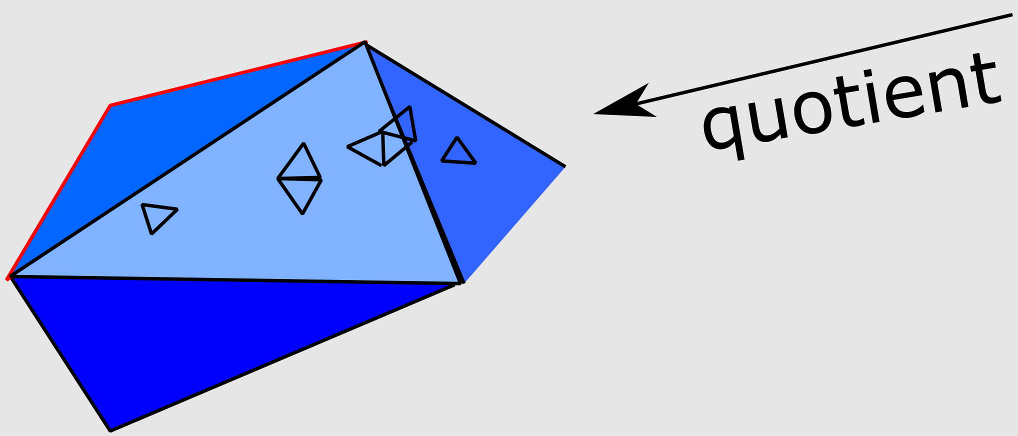
$$\eta(\psi_7) = \frac{3100+856\sqrt{7}}{259} \qquad \eta(\psi_8) = \frac{1569+370\sqrt{8}}{98}$$

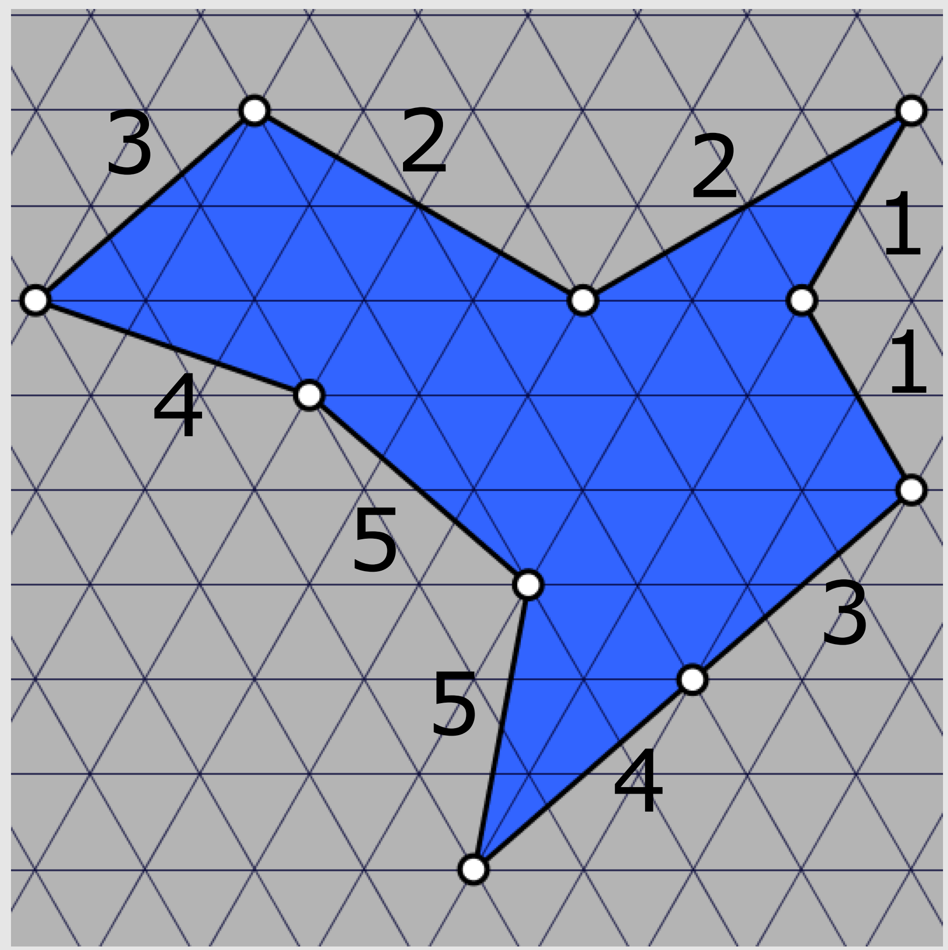
Triangulations of max degree 6.

examples:

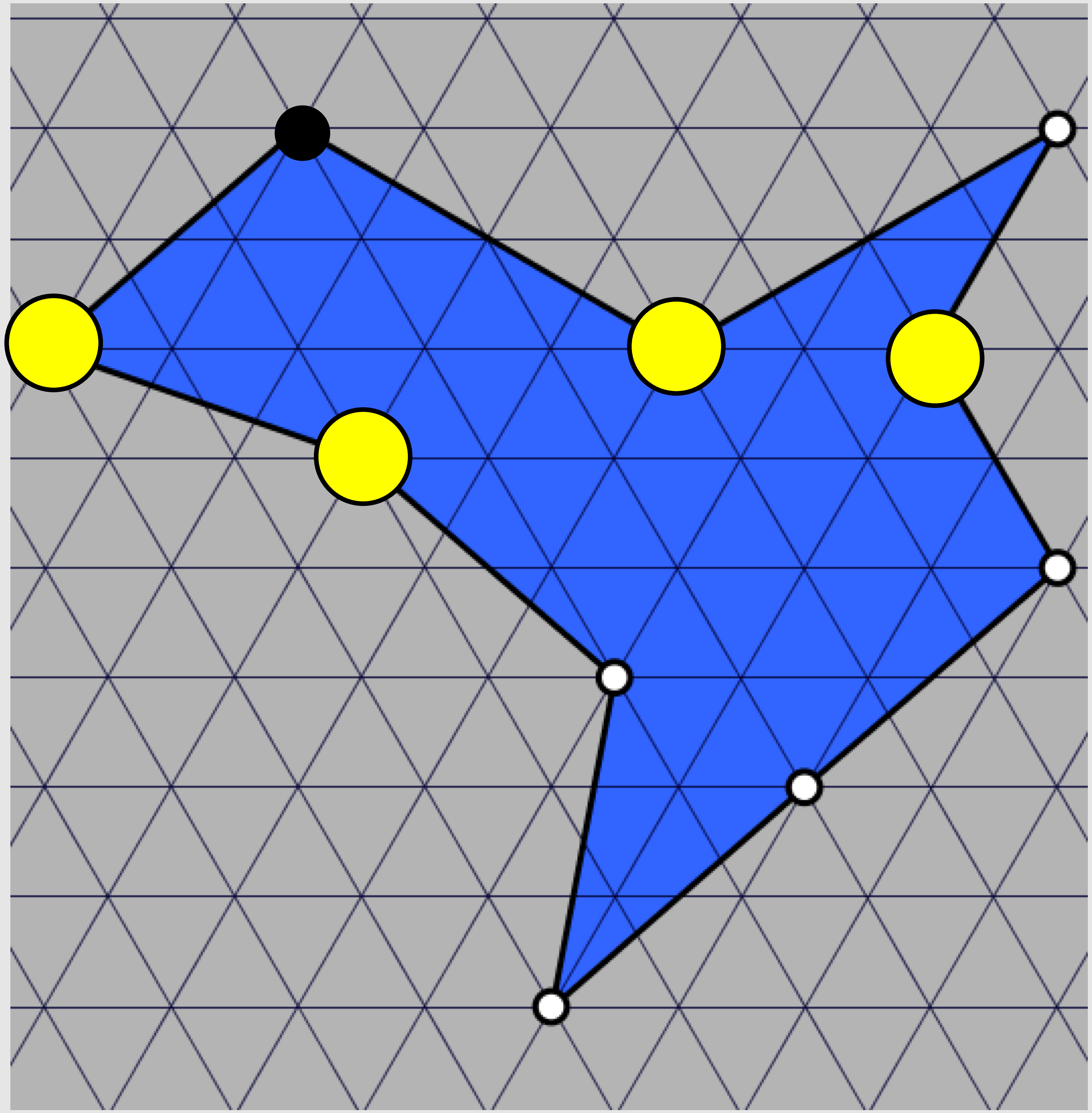


If you build the triangulation out of equilateral triangles, the resulting metric space is isometric to the flat cone manifold with at most 12 cone points of positive curvature..





Thurston coordinates



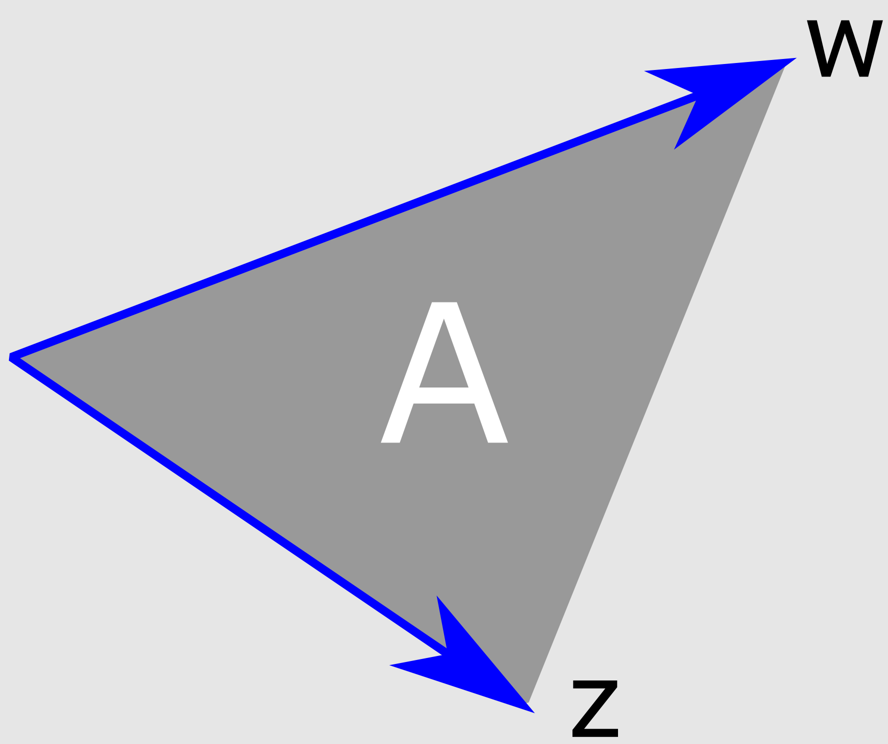
independent variables

dependent variables

origin

Gives local coords from the moduli space to C4. Triangs correspond to (EIS)^4

More structure:



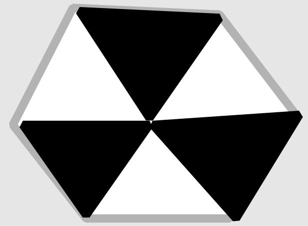
$$A = \frac{1}{4i}(z\bar{w} - w\bar{z})$$

The area of the flat cone structure in Thurston coords os the diagonal part of a Hermitian form of signature (1,n).

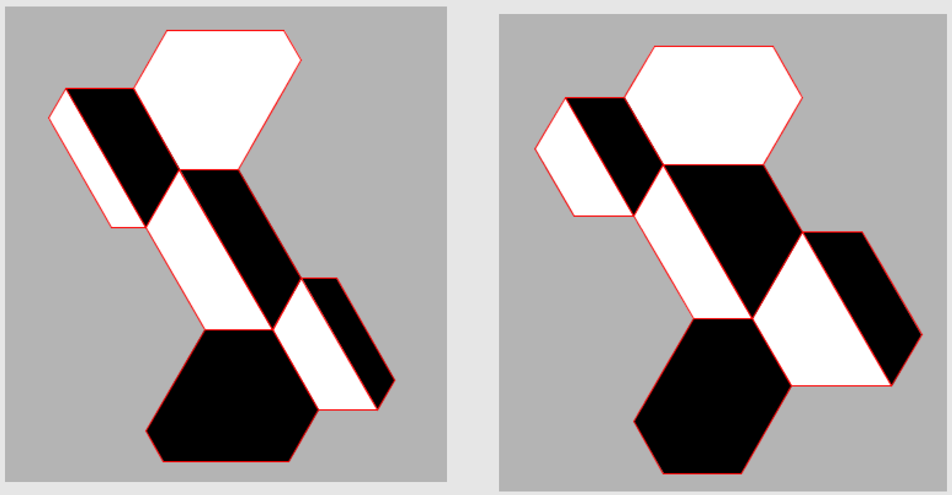
It means that globally the triangulations are "rational points" in a complex hyperbolic orbifold.

Experimental observation.

Call a 4-coloring solution generic if you never see an alternating pattern around a vertex:

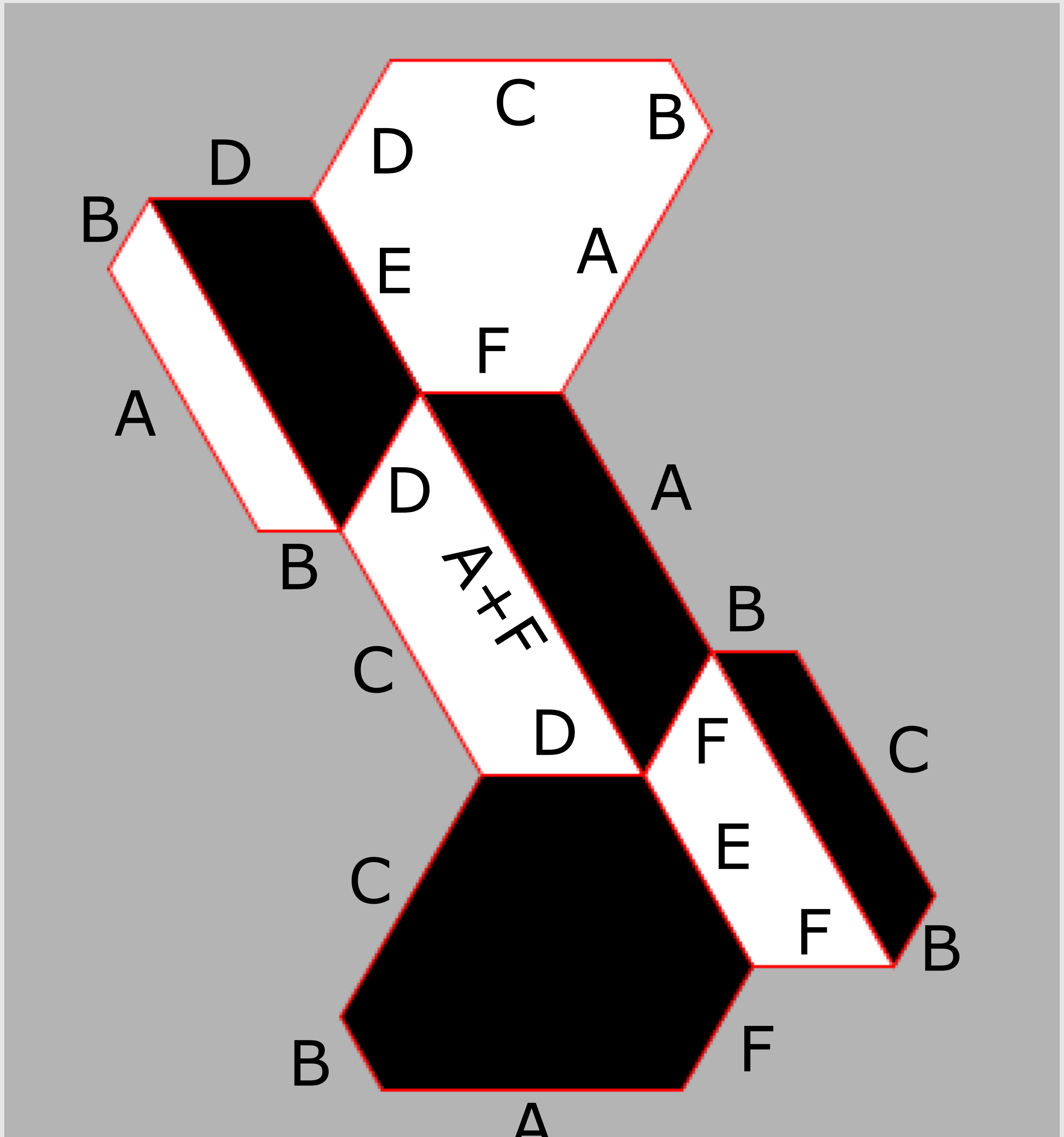


Call 2 generic solutions equivalent if they define the same shape packing up to homeomorphism.



The equivalence classes of generic solutions seem to be rational linear subspaces of the moduli space, having half the dimension of the total space.

Here's what rational subspace means:.



Compare Tutte's square packing theorem