Dr. Z.'s Math 421 Solutions to Second Chance Club Qualifying Exam, Dec. 11, 2014, 12:50-1:10pm, HLL 116

NAME: (print!) _____

1. A root of the algebraic equation $x^5 - x - 30 = 0$ is:

(a) $x = 3$ (b) $x = 2$ (c) $x = 5$ (d) $x = -1$ (e) $x = 0$ (f) $x = 10$

Ans.: **a**

2. The first three terms of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cos nx$ are

(a) $-\cos x + \frac{1}{4}\cos 2x - \frac{1}{9}\cos 3x$ (b) $-\cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$

(c) $-\cos x + \frac{1}{2}\cos 2x$ (d) $-\cos x - \frac{1}{4}\sin 2x$

Ans.: **b**

3. The solution of the system of equations

$$x + y + z = 3 \quad , \quad x + 2y + z = 4 \quad , \quad x^2 + y^2 + z^2 = 3 \quad .$$

is

(a) $x = 2, y = -1, z = 3$ (b) $x = 2, y = 1, z = -1$ (c) $x = 1, y = 1, z = 1$ (d) $x = 2, y = 0, z = 1$

Ans.: **c**

4. A solution of the ordinary differential equation $y'(t) + y(t)^2 = 2t + t^4$ is

(a) $y(t) = 5$ (b) $y(t) = -t$ (c) $y(t) = t^2$ (d) $y(t) = \sin t$

Ans.: **c**

5. The solution of the pde $u_{xx} + 2u_{yy} = 6$ in $0 \le x \le 1$, $0 \le y \le 1$, with boundary conditions $u(x, 0) = x^2, u(0, y) = 2y^2, u(x, 1) = x^2 + 2, u(1, y) = 1 + 2y^2$

(a) $u(x, y) = x^2 + y^2$ (b) $u(x, y) = x^2 + 2y^2$ (c) $u(x, y) = x^2 + 2y^2 + 5$ (d) $u(x, y) = -x^2 + 2y^4 + 5$

Ans.: **None of the above**

Note: I meant the pde to be $u_{xx} + 2u_{yy} = 10$, and then the answer would have been **b**. I gave full credit for the people who marked b, since the boundary conditions agree. I thank Christine Ray for pointing out the error in the question.

6. If $f(x) = 5$ and $g(x) = f(4x)$ then the function $g(x)$ equals

(a) 20 (b) $20x$ (c) 5 (d) $4x$

Ans.: **c**

Note: Many people answered **incorrectly** that it is **a**. **A CONSTANT FUNCTION DOES NOT DEPEND** on x, so, it stays the same no matter what you change x to.

7. If n is an integer, then a simplification of $\cos n\pi$ is

(a) 0 (b) 1 (c) $(-1)^n$ (d) -1

Ans.: **c**

7b. If a is any real number, then a simplification of $\cos a\pi$ is

(a) 0 (b) 1 (c) $(-1)^a$ (d) there is no way to simplify it further

Ans.: **d**

8. The simplified version of $\cos(x^3 - 10\pi)$ is

(a) $\cos(x^3) - \cos(10\pi)$ (b) $\sin(x^3)$ (c) $\cos(x^3)$ (d) $\cos(10\pi)$

Ans.: **c**

Note: the trig functions, $\sin x$ and $\cos x$ are **PERIODIC** of period 2π. Adding or subtractiong any multiple of 2π does not change them.

8. The simplified version of $\sin(x^3 - 11\pi)$ is

(a) $\sin(x^3) - \sin(11\pi)$ (b) $\sin(x^2)$ (c) -$\sin(x^3)$ (d) $\sin(11\pi)$

Ans.: **c**

Note: by the above, this is the same as $\sin(x^3 - \pi)$, and now use $\sin(w - \pi) = -\sin w$.

9 The Fourier Series of $f(x) = \sin x + 2\cos x - 12\sin 2x$ over $(-\pi, \pi)$ is

(a) $f(x) = \sin x + 2\cos x - 12\sin 2x$ (b) $f(x) = \sum_{n=1}^{\infty} \frac{1}{n}\cos nx$

(c) $f(x) = \sum_{n=1}^{\infty} \frac{1}{n}\sin nx$ (d) $\sin x + \cos x + \sin 2x$

Ans.: **a**

Note: If the function, defined over $(-\pi, \pi)$ is a finite combination of pure sine and/or cosine functions, its Fourier series is **itself**.

10 If $u(x,t)$ is a solution of a *specific* pde with *specific* boundary and initial conditions which of the following is **gibberish**

(a) $u(x,t) = \sin 2x \cos 4t + 5 \cos 2x \sin 5t$ (b) $u(x,t) = \sin nx \cos nt$

(c) $u(x,t) = \sum_{n=1}^{\infty} \frac{1}{n^5} \sin nx \cos 2nt$ (d) $u(x,t) = e^{-kn^2 t} \cos nx$

Ans.: **b** and **d**

Note: Without a $\sum$ in front both **b** and **d** are disembodied terms, and do not make sense.