Solutions to Dr. Z.'s Math 421(1), (Fall 2014, RU) REAL Quiz #3

1. Compute (using Lecture 4's method!) $\mathcal{L}\{t^2e^t\}$.

Sol. to 1: We are supposed to use the formula

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s) \quad ,$$

where $F(s) = \mathcal{L}\{f(t)\}$.

Here $f(t) = e^t$ so $F(s) = \frac{1}{s-1} = (s-1)^{-1}$. We have $F'(s) = (-1)(s-1)^{-2}$ and $F''(s) = (-1)(-2)(s-1)^{-3} = \frac{2}{(s-1)^3}$. Hence

$$\mathcal{L}\{t^2e^t\} = (-1)^2\frac{2}{(s-1)^3} = \frac{2}{(s-1)^3} \quad .$$

Ans. to 1: $\frac{2}{(s-1)^3}$.

2. Solve the IVP

$$y' + 2y = \delta(t-1) + 2\delta(t-2) \quad , \quad y(0) = 2 \quad .$$

Sol. to 2: Appply $\mathcal{L}$, we get

$$\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{\delta(t-1)\} + 2\{\delta(t-2)\} = e^{-s} + 2e^{-2s}$$

Putting, as usual $\mathcal{L}\{y\} = Y$, we have

$$sY - y(0) + 2Y = e^{-s} + 2e^{-2s}$$

But, since $y(0) = 2$, we have

$$sY - 2 + 2Y = e^{-s} + 2e^{-2s}$$

Solving for Y:

$$(s+2)Y = 2 + e^{-s} + 2e^{-2s} \quad ,$$

dividing by $(s+2)$:

$$Y = \frac{2}{s+2} + \frac{e^{-s}}{s+2} + 2\frac{e^{-2s}}{s+2} \quad .$$

Since $\mathcal{L}^{-1}\{\frac{1}{s+2}\} = e^{-2t}$, we get,

Ans. to 2:

$$y(t) = 2e^{-2t} + e^{-2(t-1)}\mathcal{U}(t-1) + 2e^{-2(t-2)}\mathcal{U}(t-2) \quad .$$