**Solution to Dr. Z.'s Math 421(2), REAL Quiz #2**

**1.** Use the Laplace Transform to solve the given initial-value problem.

$$y'' - 3y' + 2y = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

**Sol.**: First apply $\mathcal{L}$:

$$\mathcal{L}\{y'' - 3y' + 2y\} = 0 \quad .$$

Now $\mathcal{L}\{y''\} = s^2Y - sy(0) - y'(0) = s^2Y - 2s - 3$, $\mathcal{L}\{y'\} = sY - y(0) = sY - 2$, to get

$$\mathcal{L}\{y''\} - 3\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = s^2Y - 2s - 3 - 3(sY - 2) + 2Y = 0$$

Expanding:

$$s^2Y - 2s - 3 - 3sY + 6 + 2Y = s^2Y - 2s + 3 - 3sY + 2Y = 0 \quad .$$

Collecting terms, and moving the non-$Y$ stuff to the right side:

$$(s^2 - 3s + 2)Y = 2s - 3 \quad .$$

Solving for $Y$, we get

$$Y = \frac{2s - 3}{s^2 - 3s + 2} \quad .$$

We now need to apply $\mathcal{L}^{-1}$, but first we need to express $\frac{2s-3}{s^2-3s+2}$ in terms of partial-fractions. Factorizing

$$Y = \frac{2s - 3}{(s - 1)(s - 2)} \quad .$$

Using the **templace** $\frac{A}{s-1} + \frac{B}{s-2}$ we get:

$$Y = \frac{2s - 3}{(s - 1)(s - 2)} = \frac{A}{s - 1} + \frac{B}{s - 2} = \frac{A(s - 2) + B(s - 1)}{(s - 1)(s - 2)} \quad .$$

The denominators match automatically, so we set the numerators equal to each other

$$2s - 3 = A(s - 2) + B(s - 1) \quad .$$

Convenient values: $s = 2$ gives $2(2) - 3 = A(0) + B(2 - 1)$ so $B = 1$. $s = 1$ gives $2(1) - 3 = A(1 - 2) + B(0)$, so $-1 = -A$ and $A = 1$. Going back to the template:

$$Y = \frac{1}{s - 1} + \frac{1}{s - 2} \quad .$$

Now it is time to apply $\mathcal{L}^{-1}$:

$$y(t) = \mathcal{L}^{-1}\{\frac{1}{s - 1} + \frac{1}{s - 2}\} = e^t + e^{2t} \quad .$$

**Ans.**: $y(t) = e^t + e^{2t}$ .