Solutions to Dr. Z.'s Math 421(2) REAL Quiz #10

1. (10 points) Find the general expression, in polar coordinates, for the steady-state temperature $u(r,\theta)$ in the **infinite** plane with a circular hole of radius 2 cut-out, and where the temperature at the bounding ring is $u(2,\theta) = \cos 2\theta - 3\sin 4\theta, 0 < \theta < 2\pi$.

Sol. The boundary function $f(\theta) = u(2,\theta) = \cos 2\theta - 3\sin 4\theta$ is its own Fourier Series, so all we need is multiply each $\sin n\theta$ and/or $\cos n\theta$ by $(r/c)^{-n}$ (since this is an *infinite* plate, we need *negative powers*).

Here $c = 2$. There is one pure-cosine term, and there is one pure-sine term.

For $\cos 2\theta$ $n = 2$ so we multply it by $(r/2)^{-2}$, getting $(r/2)^{-2}\cos 2\theta{=}4r^{-2}\cos 2\theta$.

For $-3\sin 4\theta$ $n = 4$ so we multply it by $(r/2)^{-4}$, getting $-3\sin 4\theta(r/2)^{-4} = -3(\sin 4\theta)\cdot(16r^{-4}) = -48r^{-4}\sin 4\theta$

so the answer is simply

$$u(r,\theta) = 4r^{-2}\cos 2\theta - 48r^{-4}\sin 4\theta \quad .$$

Comment: Many people didn't get it. They did the related problem of a finite circular plate of ratius 2. There answer would make us all burn. Please check your answer and make sure that it makes physical sense. Don't just follow rules blindly.

What a shame! It is such an easy problem. Please review the problem and make sure you understand how to do it.