Solutions to Dr. Z.'s Math 421(1) Quiz #10

1. (10 points) Find the general expression, in polar coordinates, for the steady-state temperature $u(r,\theta)$ in the **infinite** plane with a circular hole of radius 2 cut-out, and where the temperature at the bounding ring is $u(2,\theta) = 2\sin 3\theta, 0 < \theta < 2\pi$.

Sol. The boundary function $f(\theta) = 2\sin 3\theta$ is its own Fourier Series, so all we need is multiply each $\sin n\theta$ and/or $\cos n\theta$ by $(r/c)^{-n}$ (since this is an *infinite* plate, we need *negative powers*).

Here there is only one pure-sine term, so the answer is simply

$$u(r,\theta) = 2(r/2)^{-3}\sin 3\theta = 16r^{-3}\sin 3\theta \quad .$$

Comments: Many people solved the related problem of a circular place of radius r and got, $u(r,\theta) = 2(r/2)^3\sin 3\theta = 16r^{-3}\sin 3\theta$.

READ THE QUESTION, this answers makes no physical sense, since we have an infinite place, and for large r the temperature would get so hot that it would kill us all!

Please make sure that you read the question! It is such an easy problem. Please review the problem and make sure you understand how to do it.