NAME: (print!) _____

E-Mail address: _____

MATH 421 (2), Dr. Z. , Exam I, Thurs., Oct. 9, 2014, 12:00-1:20pm, SEC 216

No Calculators! Only the official cheatsheet is allowed!
Write the final answer to each problem in the space provided. Incorrect answers (even due to minor errors) can receive at most one half partial credit, so please check and double-check your answers.

Do not write below this line (office use only)

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1. (out of 15)

2. (out of 15)

3. (out of 15)

4. (out of 15)

5. (out of 15)

6. (out of 15)

7. (out of 10)

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total: (out of 100)

1. (15 points) Use any method to compute $\mathcal{L}\{f(t)\}$ if

$$f(t) = (e^t + 1)(e^{2t} + 1) + t^3(t + 1) \quad .$$

2. (15 points) Solve the initial value problem:

$$y'' - y = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 0 \quad .$$

3a. (13 points) Solve the initial value problem

$$y' + y = f(t) \quad , \quad y(0) = 0 \quad , where \quad f(t) = \begin{cases} 1, & \text{if } 0 \le t \le 1; \\ -1, & \text{if } 1 \le t < \infty. \end{cases} \quad .$$

3b (2 points) What is $y(2)$?

4. (15 points) Solve the following integral equation

$$f(t) = te^t + \int_0^t \tau f(t-\tau)\, d\tau \quad .$$

5. (15 points) Solve the initial value problem

$$y'' + 6y' + 10y = \delta(t - \pi) \quad , \quad y(0) = 0 \quad , \quad y'(0) = 0 \quad .$$

6. (15 points) Solve the following system of differential equations with the indicated initial values.

$$\frac{dx}{dt} = -x + 5y \quad ,$$

$$\frac{dy}{dt} = -2x + y \quad ,$$

$$x(0) = 4 \quad , \quad y(0) = -2 \quad .$$

7. (10 points) Find all the eigenvalues of the matrix

$$\begin{bmatrix} 7 & -4 \\ 12 & -7 \end{bmatrix} \quad ,$$

and determine a basis for each eigenspace.