NAME: (print!) _____

E-Mail address: _____

MATH 421 (1), Dr. Z. , Exam I, Thurs., Oct. 9, 2014, 10:20-11:40am, SEC 212

No Calculators! You may only use the official Cheatsheet!
Write the final answer to each problem in the space provided. Incorrect answers (even due to minor errors) can receive at most one half partial credit, so please check and double-check your answers.

Do not write below this line (office use only)

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1. (out of 15)

2. (out of 15)

3. (out of 15)

4. (out of 15)

5. (out of 15)

6. (out of 15)

7. (out of 10)

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total: (out of 100)

1. (15 points) Using the **definition** find the Laplace transform $\mathcal{L}\{f(t)\}$ (alias $F(s)$) of

$$f(t) = \begin{cases} 1, & \text{if } 0 \le t \le 2; \\ -3, & \text{if } t \ge 2. \end{cases}$$

2. (15 points) Find

$$\mathcal{L}^{-1}\{\frac{3s^2-6s+2}{s(s-1)(s-2)}\}$$

3a. (7 points) Compute $\mathcal{L}\{(t+4)\mathcal{U}(t-4)\}$.

3b. (8 points) Compute

$$\mathcal{L}^{-1}\{\frac{e^{-2s}}{(s-2)^5}\} \quad .$$

4. (15 points) Evaluate

$$\mathcal{L}\{\int_0^t \tau^5 e^{t-\tau}\, d\tau\} \quad .$$

5. (15 points) Solve the initial-value problem

$$y'' + 2y' + y = \delta(t-3) \quad , \quad y(0) = 0 \quad , \quad y'(0) = 0 \quad .$$

6. (15 points) Solve the system of ordinary differential equations with the given initial values

$$\frac{dx}{dt} = -x + y \quad ,$$

$$\frac{dy}{dt} = 2x \quad ,$$

$$x(0) = 0 \quad ; \quad y(0) = 2 \quad .$$

7. (10 points) Find all the eigenvalues of the matrix

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \quad ,$$

and determine a basis for each eigenspace.