Dr. Z.'s Calc5 Homework assignment 3

In 1-3, Compute the Laplace Transforms

1. $\mathcal{L}\{t^5 e^{-2t}\}$.

2. $\mathcal{L}\{e^{3t}\sin 2t\}$.

3. $\mathcal{L}\{e^{4t}\cos 2t - t^2 e^{2t}\}$.

In 4-5 compute the inverse Laplace Transforms

4.
$$\mathcal{L}^{-1}\{\frac{1}{s^2-2s+10}\} \quad .$$

5.
$$\mathcal{L}^{-1}\{\frac{2s+1}{s^2-6s+10}\} \quad .$$

6. Use the Laplace Transform to solve the following initial-value problem.
$$y' - y = e^t \quad , \quad y(0) = -1.$$

7. Use the Laplace Transform to solve the following initial-value problem.
$$y'' - 2y' + y = t^2 e^t \quad , \quad y(0) = 0 \quad , \quad y'(0) = 0 \quad .$$

8. Express in terms of $\mathcal{U}(t-a)$ the following function
$$f(t) = \begin{cases} t+1, & \text{if } 0 \le t < 3; \\ 0, & \text{if } t \ge 3. \end{cases} \quad ,$$

9. Compute
$$\mathcal{L}\{t^2\, \mathcal{U}(t-3)\} \quad .$$

10. Compute
$$\mathcal{L}\{\sin t\, \mathcal{U}(t-\pi/2)\} \quad .$$

11. Compute
$$\mathcal{L}^{-1}\{\frac{e^{-2s}}{s^4}\} \quad .$$

12. Compute
$$\mathcal{L}^{-1}\{\frac{e^{-3s}}{(s-1)^2}\} \quad .$$

13. Solve the initial value problem:
$$y' - 2y = f(t) \quad , \quad y(0) = 0 \quad , where \quad f(t) = \begin{cases} 2, & \text{if } 0 \le t < 1; \\ -1, & \text{if } 1 \le t < \infty. \end{cases} \quad ,$$