Dr. Z.'s Calc5 Homework assignment 12

1. The **Chebyshev** polynomials of the first kind, $T_n(x)$ are defined by

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

with initial conditions $T_0(x) = 1, T_1(x) = x$. Find $T_2(x)$,$T_3(x)$, and $T_4(x)$.

2. The **Laguere** polynomials $L_n(x)$ are defined by

$$L_n(x) = \frac{(2n-1)-x}{n} L_{n-1}(x) - \frac{n-1}{n} L_{n-2}(x)$$

with initial conditions $L_0(x) = 1, L_1(x) = 1 - x$. Find $L_2(x)$,$L_3(x)$, and $L_4(x)$.

3. The **Zeilberger** polynomials $Z_n(x)$ are defined by

$$Z_n(x) = n^3x^2Z_{n-1}(x) - (n+1)Z_{n-2}(x) + n^2Z_{n-3}(x)$$

with initial conditions $Z_0(x) = 1, Z_1(x) = x, Z_2(x) = x^2$. Find $Z_3(x)$ and $Z_4(x)$.

4. Find the first four coefficients of the Fourier-Legendre expansion of

$$f(x) = \begin{cases} -1, & \text{if } -1 < x < 0; \\ 0, & \text{if } 0 \leq x < 1. \end{cases}$$

5. Find the **full** Fourier-Legendre expansion of $f(x) = x^3$ on $(-1, 1)$.

6. Find the first three coefficients of the Fourier-Legendre expansion of $f(x) = e^x$ on $(-1, 1)$.