Dr. Z.'s Calc5 Homework assignment 0

1. Approximate, with mesh-size $h = 1$, the solution of the boundary-value problem

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < 2 \quad , \quad 0 < y < 2 \quad ;$$

subject to the boundary conditions

$$u(0, y) = 1 \quad , \quad 0 < y < 2 \quad ; \quad u(2, y) = 2 \quad , \quad 0 < y < 2 \quad ;$$

$$u(x, 0) = -1 \quad , \quad 0 < x < 2 \quad ; \quad u(x, 2) = 3 \quad , \quad 0 < x < 2 \quad .$$

2. Approximate, with mesh-size $h = 2$, the solution of the boundary-value problem

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < 4 \quad , \quad 0 < y < 4 \quad ;$$

subject to the boundary conditions

$$u(0, y) = y^2 \quad , \quad 0 < y < 4 \quad ; \quad u(4, y) = y^3 \quad , \quad 0 < y < 4 \quad ;$$

$$u(x, 0) = 2x \quad , \quad 0 < x < 4 \quad ; \quad u(x, 4) = 1 - x \quad , \quad 0 < x < 4 \quad .$$

3. Approximate, with mesh-size $h = \frac{1}{2}$, the solution of the boundary-value problem

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < \frac{3}{2} \quad , \quad 0 < y < \frac{3}{2} \quad ;$$

subject to the boundary conditions

$$u(0, y) = y \quad , \quad 0 < y < \frac{3}{2} \quad ; \quad u(\frac{3}{2}, y) = 1 + y \quad , \quad 0 < y < \frac{3}{2} \quad ;$$

$$u(x, 0) = 1 - x \quad , \quad 0 < x < \frac{3}{2} \quad ; \quad u(x, \frac{3}{2}) = 1 + x \quad , \quad 0 < x < \frac{3}{2} \quad .$$

4. Set-up, but do not solve, the system for approximating, with mesh-size $h = \frac{1}{4}$, the solution of the boundary-value problem

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < 1 \quad , \quad 0 < y < 1 \quad ;$$

subject to the boundary conditions

$$u(0, y) = y \quad , \quad 0 < y < 1 \quad ; \quad u(1, y) = -y \quad , \quad 0 < y < 1 \quad ;$$

$$u(x, 0) = 1 \quad , \quad 0 < x < 1 \quad ; \quad u(x, 1) = -x \quad , \quad 0 < x < 1 \quad .$$