NAME: (print!) _____

E-Mail address: _____

MATH 421 (1), Dr. Z. , FINAL EXAM, Mon., Dec. 22, 2014, 8:00-11:00am, SEC 212

No Calculators!, You can only use the official "cheatsheet" downloaded from
http://www.math.rutgers.edu/~zeilberg/calc5/cheatsheet.pdf
Write the final answer to each problem in the space provided. Incorrect answers (even due to minor errors) can receive at most one half partial credit, so please check and double-check your answers.

Do not write below this line (office use only)

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1. (out of 15)

2. (out of 15)

3. (out of 15)

4. (out of 15)

5. (out of 15)

6. (out of 15)

7. (out of 15)

8. (out of 15)

9. (out of 15)

10. (out of 15)

11. (out of 15)

12. (out of 15)

13. (out of 10)

14. (out of 10)

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total (out of 200)

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1. (15 pts.) Find the general expression, in polar coordinates, for the steady-state temperature $u(r, \theta)$ in an **infinite** plate with a circular hole, centered at the origin, of radius 2, if the temperature on $r = 2$ is given by $u(2, \theta) = -7 \cos \theta + \frac{1}{4} \sin 5\theta$.

Ans.: $u(r, \theta) =$

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2. (15 points altogether) (a) (8 pts.) Show that the set of functions $\{\cos(n\pi x)\}$, $n = 1, 2, \ldots$ is orthogonal over the interval $[0, 1]$.
(b)(7 pts.) Find the norm of each function.

Ans. to (b): $||\cos(n\pi x)|| =$

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3. (15 points) Find the **complex** Fourier series of the function $f(x) = x+5$ on the interval $-1 < x < 1$.
(Warning: The interval in this problem is **not** $-\pi < x < \pi$).

Ans.:

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4. (15 points) The **Kermit** polynomials $K_n(x)$ are defined by

$$K_n(x) = K_{n-1}(x) + K_{n-2}(x) + nxK_{n-3}$$

with initial conditions $K_0(x) = 1, K_1(x) = 1, K_2(x) = 1$. Find $K_4(x)$.

Ans.: $K_4(x) =$

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5. (15 points) Solve the boundary value pde problem:

$$16u_{xx} = u_{tt} \quad , 0 < x < \pi \quad , \quad t > 0 \quad ;$$

$$u(0,t) = 0 \quad , \quad u(\pi,t) = 0 \quad , \quad t > 0 \quad ;$$

$$u(x,0) = \sin x + \sin 2x \quad , \quad u_t(x,0) = 20\sin 3x - 4\sin 4x \quad , \quad 0 < x < \pi \quad .$$

Ans: $u(x,t) =$

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6. (15 points) Find the Fourier series of $f(x) = 10 + \cos x + 2\cos 3x + 6\sin x + 12\sin 3x$ over the interval $(-\pi, \pi)$.

Ans.:

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7. (15 points) Set up the boundary value problem for the steady-state temperature $u(x, y)$, where a thin rectangular plate coincides with the region in the xy-plane defined by $0 \leq x \leq 30, 0 \leq y \leq 40$ if the bottom and right sides are insulated, while the top side is held at temperature 200 and the left side is held at temperature 100.

Ans.: pde:

boundary conditions:

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8. (15 points) Use any method to compute $\mathcal{L}\{f(t)\}$ if

$$f(t) = (e^t + 1)(e^t - 1) + (t + 2)(t - 2) \quad .$$

Ans.:

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9. (15 points) Use the **Laplace transform** (no credit for other methods!) to solve the pde

$$u_{xx} = u_{tt} \quad , 0 < x < \pi \quad , \quad t > 0 \quad ,$$

subject to the **boundary-conditions**

$$u_x(0,t) = 0 \quad , \quad u_x(\pi,t) = 0 \quad , \quad t > 0 \quad ,$$

and the **initial conditions**

$$u(x,0) = \cos x \quad , \quad u_t(x,0) = 0 \quad , \quad 0 < x < \pi \quad .$$

Ans.:

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10. (15 points) Solve the following integral equation

$$f(t) = 4e^t + \int_0^t \tau f(t-\tau)\, d\tau \quad .$$

Ans.:

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11. (15 points) Solve the initial value problem

$$y'' + 2y' + 2y = \delta(t - \pi) \quad , \quad y(0) = 0 \quad , \quad y'(0) = 0 \quad .$$

Ans.:

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12. (15 points) Find the Fourier integral representation of

$$f(x) = \begin{cases} 0, & \text{if } x < -\pi; \\ 1, & \text{if } -\pi \le x < 0; \\ 2, & \text{if } 0 \le x \le \pi; \\ 0, & \text{if } x > \pi; \end{cases}$$

Ans.:

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13. (10 points) Approximate, with mesh-size $h = 1$, the solution of the boundary-value problem

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < 2 \quad , \quad 0 < y < 2 \quad ;$$

subject to the boundary conditions

$$u(0, y) = 1 \quad , \quad 0 < y < 2 \quad ; \quad u(2, y) = 4 \quad , \quad 0 < y < 2 \quad ;$$

$$u(x, 0) = 2 \quad , \quad 0 < x < 2 \quad ; \quad u(x, 2) = -2 \quad , \quad 0 < x < 2 \quad .$$

Ans.: The approximation of $u(1, 1)$ is:

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14. (10 points) Find all the eigenvalues of the matrix

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \quad ,$$

and determine a basis for each eigenspace.

Ans.: Smallest eigenvalue: Corresponding eigenvector:

Largest eigenvalue: Corresponding eigenvector: